question in which angle the body is stable was not discussed. The trial and error and methods can be used however, another methods can be used. For example, the extruded rectangular has moment of inertia which is increases with the inclination angle, β $(1/\cos^3\theta)$. In some situation as depicted in the figures the inclination change to some degree or point and no further. To sharpener the question, again more tilding the rectangular results in larger moment of inertia while the volume remain the same. Thus, according to this argument all the body should continue to rotate maximum moment of inertia. In fact, the experiments that this author has done show that body is stable around that several points and they not continues (with the exception of cylinder "type" on the side). The reason for singularly of these points is that another condition has to be fulfilled. The buoyancy centerid must be on the same line vertical line with the buoyancy centerid. So the question should be locate all the angle in which the buoyancy and gravity centerids are on the same line. Then check if that point is stable. For instance, in the case of rectangular and case of $\rho_s/\rho_e ll$ is the same line are on angles $0°$ and $45°$.

Example 8.7: Moment with Angel **Level: Intermediate**

Express the moment of inertia for rectangular body as a function of the inclination. Start with complete vertical position. Assume that the trapezoid shape remains through process. Note that the moment of inertia of rectangular is $bh^3/12$.

Solution

The surface has depth of s into the page and the width of the surface is $b/\cos\theta$. The location of $\boldsymbol{\mathcal{A}}$ is independent of inclination. The buoyancy centroid at B can

$$\frac{\Delta y}{b} = \left(\frac{b}{d}\right)^2 \frac{\tan^2\theta}{12} \tag{8.7.a}$$

$$\frac{\Delta y}{b} = \left(\frac{b}{d}\right) \frac{\tan\theta}{6} \tag{8.7.b}$$

$$\alpha = \tan^{-1}\left(\frac{2\,\tan\theta}{6\left(\frac{d}{b}\right) - \tan^2\theta}\right) \tag{8.7.c}$$

The location of buoyancy centroid is at

$$\boldsymbol{GB}' = \sqrt{\left(\frac{d}{2} - \Delta y\right)^2 + (\Delta x)^2} \tag{8.7.d}$$

The critical length $\boldsymbol{GB}''$ is obtained by

$$\boldsymbol{GB}'' = \boldsymbol{GB}'\, cos\,|\alpha - \theta| \tag{8.7.e}$$

The moment of inertia can be expressed as

$$I_{xx} = \frac{s\,b^3}{12\cos^3\theta} \tag{8.7.f}$$

Example 8.8: Two triangles attached **Level: Easy**

One way to make a ship to be more hydrodynamical is by making the body as narrow as possible. Suppose that two opposite sides triangle (prism) is attached to each other to create a long "ship" see Fig. 8.23. Supposed that $\mathrm{b/h} \longrightarrow\sim 0$ the body will be unstable. On the other side if the $\mathrm{b/h} \longrightarrow\sim \infty$ the body is very stable. What is the minimum ratio of $\mathrm{b/h}$ that keep the body stable at half of the volume in liquid (water). Assume that density ratio is ρ_s/ρ_ℓ. Is there limits on the density ratio.

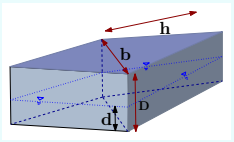


Fig. 8.23 – Stability of two triangles put together.

Solution

The answer the question requires solve the governing equation. In this case the moment of inertia of half triangle is $hb^3/24$ for rotating triangle around the base. There are four half triangles and thus the total moment of inertia is $hb^3/6$. The immersed volume of the body is

$$V_0 = 2\left(\frac{b\,h\,d}{2}\right) = b\,h\,d$$

The relationship of the density ratio to heights (Archimedes Law) is

$$d = \frac{D\,\rho_s}{\rho_\ell} \tag{8.5}$$

The governing equation is

$$\frac{D}{2} - \frac{d}{2} = \frac{\cancel{b^2}\,\cancel{h}}{6\,\cancel{b}\,\cancel{h}\,d} \longrightarrow \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) = \frac{1}{6}\,\frac{b^2}{d}$$

or in a dimensional form as

$$\left(1 - \frac{\rho_s}{\rho_\ell}\right)\frac{\rho_s}{\rho_\ell} = \frac{1}{3}\left(\frac{b}{D}\right)^2 \tag{8.6}$$

According to Eq. (8.6) the stability is affected by the density ratio and by the ratio of b/D. Furthermore, according to this model the stability is not function of length, h. However, according to this model it is build on the idea that that there in marine terminology there is no pitch (rotating in the other direction). If h is the same order as b then this model is fail to take into account the stability of the other direction. Hence, with these hand waving argument h must at least two time of b.

End of Ex. 8.8

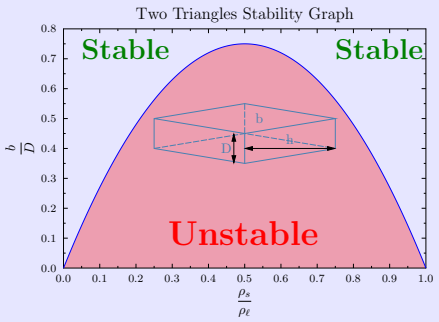


Fig. 8.24 – Two attached triangles stability diagram. Shown that minimum b is at least in the same size of D.

Example 8.9: Extruded Triangle angle

Level: Intermediate

This example is expansion the question 3.4 by Lautrup (Lautrup 2011). Extruded triangle with a shape pointed point down is placed in a liquid. When alpha angle is very sharp the triangle is not very stable. On the other hand, when the angle is very wide the triangle is stable. Assuming that the area change does not affect the analysis. what is the minimum angle for with the body is stable for arbitrary density ratio ($0 \leq \rho_r \leq 1$). This example only provide the trends and it sufficient of the make of the point of the range where to expect the stable zone.

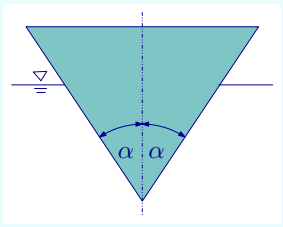


Fig. 8.25 – Triangle With The Peak Down stability analysis.

Solution

The Symmetry dictated that it is sufficient to balance half the triangle. The density ratio related to the draft as

$$\rho_s \frac{D\, D \tan\alpha}{2} = \rho_\ell \frac{d\, d \tan\alpha}{2} \longrightarrow \frac{\rho_s}{\rho_\ell} = \left(\frac{d}{D}\right)^2 \tag{8.9.a}$$

The moment of inertia of the of the cross section area is

$$I_{xx} = \frac{s\,(2\,d\,\tan\alpha)^3}{12} \tag{8.9.b}$$

End of Ex. 8.9

The volume is

$$V_0 = \frac{\cancel{2}\, s\, d^2 \tan\alpha}{\cancel{2}} \longrightarrow V_0 = s\, d^2 \tan\alpha \tag{8.9.c}$$

The governing equation is

$$G - B = \frac{I_{xx}}{V_0} = \frac{\frac{2\cancel{s}\,(d\,\tan\alpha)^3}{3}}{\cancel{s}\, d^2 \tan\alpha} = \frac{2\,d\,\tan^2\alpha}{3} \tag{8.9.d}$$

The distance $G - B$ the difference between the two triangles. Hence

$$\frac{2\,D}{3} - \frac{2\,d}{3} = \frac{2\,d\,\tan^2\alpha}{3} \longrightarrow \left(1 - \frac{d}{D}\right) = \frac{d}{D}\,\tan^2\alpha \tag{8.9.e}$$

The value of limit α is

$$\alpha = \tan^{-1}\sqrt{\left(1 - \frac{d}{D}\right)\frac{D}{d}} \tag{8.9.f}$$

This results is exhibited in the following graph. It is caution that Eq. (8.9.f) represents the trends only and additional correction has to be taken into account.

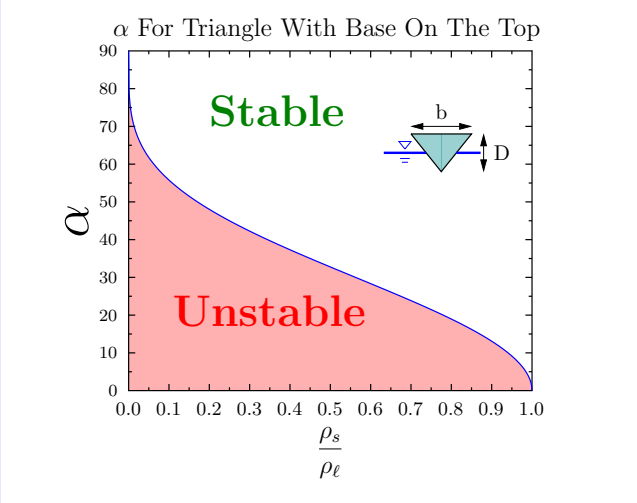


Fig. 8.26 – α as function of density ratio.

It can be observed fromFig. 8.26 that for small density ratios a large angle is required. This results is obvious and it is based on intuitive. However, it expect that required angle will have "jump" at the initial and the final zone. For large range the change is approaching a linear. Again, it must be noted that this results is not the final because the change of the rotation line and large tilde angle is applied then the triangle might not be stale.

8.4.1 Usage of Stability to find Centroid

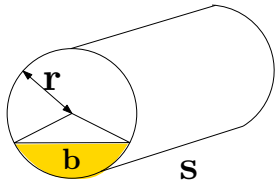


Fig. 8.27 – Circular shape for finding centroid.

The stability arguments can be used to find the centroid of fraction of circle. A cylinder is naturally stable and no matter what the value of the density ratio (location of liquid level). Hence, this calculation is based on this concept/idea, the extruded circle must obey the stability equation equation as

$$\frac{I_{xx}}{V_0} = G - B \tag{8.7}$$

Since G is fix for all circles while B is unknown. The left side of the equation can be reduced. For example, for $\rho_s/\rho_\ell = 0.5$ it becomes

$$\frac{I_{xx}}{V_0} = \frac{\frac{\cancel{8}(2r)^3}{12}}{\frac{\cancel{8}\pi\, r^2}{2}} = \frac{4\,r}{3\,\pi} \tag{8.8}$$

Thus, the centroid is found and the depth (s) is irrelevant but to the results but is useful tool in calculation the centroid of any fraction of the circle. In general, the moment of inertia of and area of the golden area can be evaluated and the direct expression is

$$\mathcal{B} = \boldsymbol{G} - \frac{I_{xx}}{V_0} = \frac{r}{2} - \frac{b^3/12}{A} \tag{8.9}$$

The area is readily available and hence center of buoyancy can be calculated.

Example 8.10: Centeroid of Quarter Segment **Level: Easy**

Calcuate the centeroid of the circular segment with the height of x as shown in the Fig. 8.28. In particular calculate the case where $x = r$. Notice that cylinder is imaginary and it should used to calcualte the moment of inertia and volume.

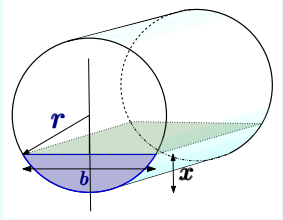


Fig. 8.28 – Centeroid of circular shape. Notice that cylinder is used to calculate Circular shape centrod (transpert dark blue).

Solution

End of Ex. 8.10

The equation to calcuate the centeroid is

$$y_c = r - \frac{(I_{xx}/s)}{A} \tag{8.10.a}$$

The moment of inertia of line (actually the green area) is $I/s = (b)^3/12$ and the area

$$A = \frac{r^2}{2}\left(sin^{-1}\left(\frac{r-x}{r}\right) - \frac{r-x}{r}\right)$$

The relationship betten b and x is based on Pythagoras theorm as

$$b = 2\sqrt{r^2 - (r-x)^2} = 2\sqrt{2\,x\,r - x^2}$$

with the given properties the centeroid can be determined.

$$y_c = r - \frac{\frac{8\left(2\,x\,r - x^2\right)^{3/2}}{12}}{\frac{r^2}{2}\left(sin^{-1}\left(\frac{r-x}{r}\right) - \frac{r-x}{r}\right)} \tag{8.10.b}$$

or in dimensionless from by introducing $\tilde{x} = x/r$ to be

$$y_c = r - \frac{\frac{8\cancel{r^3}\left(2\,\tilde{x} - \tilde{x}^2\right)^{3/2}}{12}}{\frac{\cancel{r^2}}{2}\left(sin^{-1}\left(1-\tilde{x}\right) - \left(1-\tilde{x}\right)\right)} = r\left(1 - \frac{4\left(2\,\tilde{x} - \tilde{x}^2\right)^{3/2}}{3\left(sin^{-1}\left(1-\tilde{x}\right) - \left(1-\tilde{x}\right)\right)}\right) \tag{8.10.c}$$

and the equation can be dimensionless as

$$\frac{y_c}{r} = 1 - \frac{4\left(2\,\tilde{x} - \tilde{x}^2\right)^{3/2}}{3\left(sin^{-1}\left(1-\tilde{x}\right) - \left(1-\tilde{x}\right)\right)} \tag{8.10.d}$$

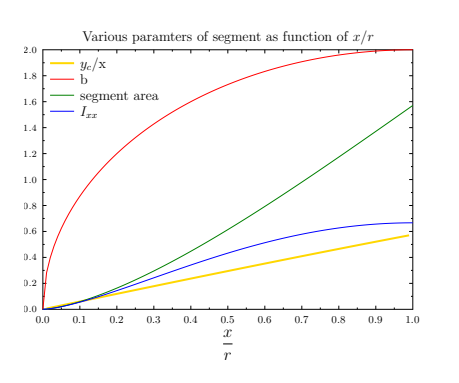


Fig. 8.29 – Various dimemnsions relate to segment.

The end values can be observed easliy. When $\tilde{x}$ is equal to one b must be 2 etc.

The question that must follow this discussion is whether the circular contour is some kind of boundary between stable and unstable contours? The investigation so far shows and indicants that it is the case but there should additional stipulation which was not formulated yet. For example, a removal of small section of the circle make the body unstable and the "circle" will turn away.

8.5 Potential Energy Approach

This method was suggested by Erdös at el and was slightly improved by Abolhassani. This method based on the idea that a derivative of potential energy can provide a location or locations where a system has a minimum (or maximum) and thus it is potential of stability point[6]. The energy used in this scenario is the gravitational energy that is expressed as

$$U_{sys} = (M + m)\, g\, h_{M+m} = g\ (m\, h_m + M\, h_M) \tag{8.10}$$

Where subscript sys referred to the entire system. The m is referred to floating body and M is referred to the displaced liquid in other words to the mass if the liquid was filling the submerged volume. The logic to the last definition is that it represents the potential of the buoyancy force acting in the center immersed part. The change in the potential is due to the change in the angle

$$\frac{dU_{sys}}{d\theta} = 0 \tag{8.11}$$

The condition that angle, θ is by checking the second derivative if it positive or negative. In away doing example it will repetitive of the moment method converting it to potential and going over the mathematics. This book is more focus on the physics and therefore it not presented.

[6] This topic should be discussed elementary physics class and not fluid mechanics textbook. However, if there will be a significant request it will be briefly discussed.

9 Metacenter

9.1 Introduction

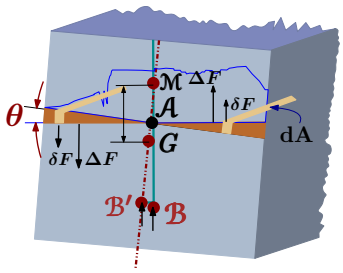


Fig. 9.1 – Stability analysis of floating body.

The last chapter two methods were discussed: the Direct Examination and the abstract Potential Energy. The cousin of the direct examination is the Metacenter method. In Metacenter method, the ship or the floating body is given a small rotation or tilde (as in Direct Examination) and instead of resulting angle, a distance from the gravity centroid to new imaginary location (Metacenter) is measured. If the distance $\boldsymbol{G}$ to $\mathfrak{M}$ is point up it considered to possible and vice versa. If the distance is positive then the body is stable and if it is negative the body is unstable. The advantages and short falls will be discussed at the end of the Chapter.

Assuming the general body is floating and it is at a certain configuration. The body is tilted by a given small angle, θ, and the resulting distance is measured. As opposed the perceived public opinion, the tilting point is around point **A** and not around imaginary point $\mathfrak{M}$[1]. If the body will be rotating around point $\mathfrak{M}$ it will not keep the displaced volume constant. The immersed part of the body centroid translates to a new location, $\mathfrak{B}'$ as shown in Fig. 9.1. The mass (gravity) centroid is still at the same old location since the body itself does not change (on moving gravity center later). This deviation of the buoyant centroid from the old buoyant centroid, $\mathfrak{B}$, is similarly as it was done in the Direct Examination. The right brown area (volume) in Fig. 9.1 is displaced by the same area (really the volume) on left since the weight of the body didn't change[2]so the total immersed area (volume) is constant.

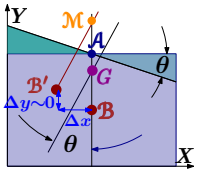


Fig. 9.2 – Description to demonstrate what is Metacenter.

In the previous chapter it was shown that Δx or $\mathfrak{B}\mathfrak{B}'$ is

$$\mathfrak{B}\mathfrak{B}' = \frac{I_{xx}\,\tan\theta}{V_0} \tag{9.1}$$

the point where the gravity force direction is intersecting with the center line of the cross section is referred as Metacenter point, $\mathfrak{M}$. The location of the Metacenter point can be obtained from the geometry as Metacenter point! definition

$$\overline{\mathfrak{B}\mathfrak{M}} = \frac{\overline{\mathfrak{B}\mathfrak{B}'}}{\sin\theta} \tag{9.2}$$

And combining equations (9.1) with (9.2) yields

$$\overline{\mathfrak{B}\mathfrak{M}} = \frac{I_{xx}\,\tan\theta}{V_0\,\sin\theta} \tag{9.3}$$

For small angle ($\theta \sim 0$)

$$\lim_{\theta\to 0}\frac{\sin\theta}{\tan\theta} \sim 1 \tag{9.4}$$

[1]For fixed $\mathcal{A}$ point.

[2]It is correct to state: area only when the body is extruded. However, when the body is not extruded, the analysis is still correct because the volume and not the area should be used.

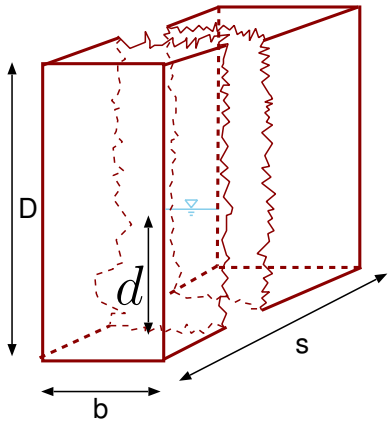


Fig. 9.3 – Cubic body dimensions for stability analysis.

It is remarkable that the results is independent of the angle. Looking at Fig. 9.2, the geometrical quantities[3] can be related as

$$\overline{GM} = \frac{\overbrace{I_{xx}}^{\overline{BM}}}{V_0} - \overline{GB} \tag{9.5}$$

Eq. (9.5) can also written for solid body as

$$\overline{GM} = \frac{\overbrace{\rho_\ell\, I_{xx}}^{\overline{BM}}}{\rho_s\, V_{body}} - \overline{GB} \tag{9.6}$$

To understand these principles consider the following examples.

9.2 Application of $\overline{GM}$

All the terms in Eq. (9.5) normally provided and it is simply plugging them into the Eq. (9.5) and obtaining the results. Illustrate these points an extensive example is provided.

Example 9.1: Cylinder Wedge **Level: Intermediate**

In Fig. 9.3 depicts the extruded rectangular with various dimensions. Assume that the body is solid with density below the liquid density, calculate the $\overline{GM}$ for various dimensions.

[3] Alternative explanation will be provided at the end of the chapter.

End of Ex. 9.1

Solution

The governing equation is

$$\boldsymbol{GM} = \frac{I_{xx}}{V_0} - \overline{\boldsymbol{GB}} \tag{9.1.a}$$

As before the densities is used to related

$$V_0\,\rho_\ell = V_{body}\,\rho_s \longrightarrow d\,\rho_\ell = D\,\rho_s \tag{9.1.b}$$

Point $\boldsymbol{G}$ is located at $D/2$ and point $\boldsymbol{\mathcal{B}}$ is located at $d/2$. Moment of inertia is $I = b^3\,s/12$ and the volume is $V_0 = d\,s\,b$ Armed with these data Eq. (9.1.a) becomes

$$\boldsymbol{GM} = \frac{\frac{b^3 \cancel{s}}{12}}{d \cancel{s} b} - \left(\frac{D}{2} - \frac{d}{2}\right) \tag{9.1.c}$$

or in dimensionless form as

$$\frac{\boldsymbol{GM}}{D} = \frac{1}{12}\left(\frac{b}{D}\right)^2\left(\frac{\rho_\ell}{\rho_s}\right) - \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{9.1.d}$$

Plotting the results of various density and b/D provides the following figure

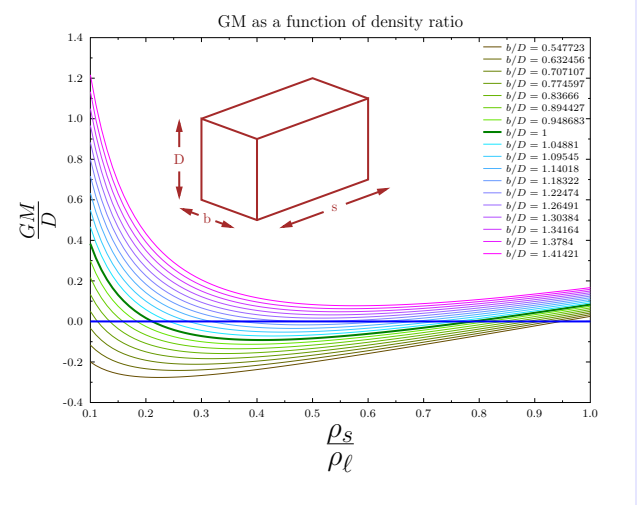


Fig. 9.4 – $\boldsymbol{GM}$ of Rectangular shape with various dimensions.

The rectangular has larger $\boldsymbol{GM}$ when floating on very heavy liquid. It is more stable if it is lighter. The blue line differentiate between positive and to negative $\overline{\boldsymbol{GM}}$ values The Fig. 9.4 exhibits the $\overline{\boldsymbol{GM}}$ as function of the density ratio for various ratio of b/D. The figure demonstrates that there is a minimum with every graph that is around the $\rho_s = 0.5\rho_\ell$. For some ratios of b/D the figure demonstrates that $\overline{\boldsymbol{GM}}$ is negative. As solid density approaches to liquid density, the body becomes more stable and even with positive $\overline{\boldsymbol{GM}}$ for some b/D ratios. At mid range density range the body is less stable.

Example 9.2: What to do to save the Ship Level: Easy

Assume that you are on a floating body (boat or ship) and it is about turn to it the side. what should you in order to save the floating body? Throw items over board or bring more things to ship like your raft that is normally tied to your boat?

Solution

If the ship or the boat is light that throwing items will make more stable. On the other the boat is almost full and you should add more items and make it as heavy as you can (even pump water into the ship). It is common to have a maximum load marking on the ship or boat. Normally this point should be design in about 30% of the ship displacement. Thus, if the convention is applied that it better to throw as much as possible. The reason that maximum mark exist is or should be for stability reasons. Load about that point will the ship unstable (below safety factor).
As anecdote of this author, on his ship mechanic duty exam (on a missile boat) a common question was what to do when ship shows signs of turning. The proper answer was to pump and throw overboard everything as possible out. The question was originated by someone experienced it first hand without any the theoretical understanding.

Example 9.3: $\mathcal{GM}$ of cylinder Level: Easy

A cylinder with a radius, r and a length D is floating on a liquid. Calculate the $\overline{\boldsymbol{GM}}$ for various densities ratios and ratios of r/D.

Solution

This example basically repeat Ex. 9.1 for cylinder. The immersed volume is $\pi\, d\, r^2$ The moment of inertia of circular shape is $\pi\, r^4/4$.

$$\boldsymbol{GM} = \frac{\cancel{\pi}\, r^4/4}{\cancel{\pi}\, d\, r^2} - \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{9..a}$$

or in a clear form

$$\boldsymbol{GM} = \frac{r^2}{4\, d} - \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{9..b}$$

Or in a dimensionless form as

$$\frac{\boldsymbol{GM}}{D} = \frac{1}{4}\left(\frac{r}{D}\right)^2 \frac{\rho_\ell}{\rho_s} - \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{9..c}$$

End of Ex. 9.3

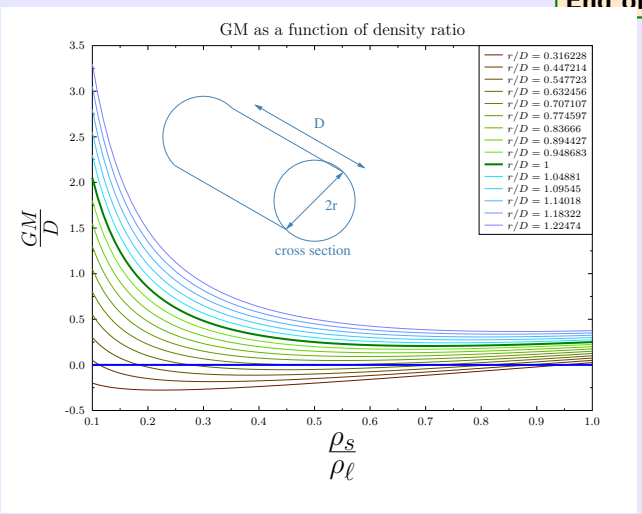


Fig. 9.5 – $\overline{GM}$ as a function of density ratio for various r/D.

9.2.1 Unstable Bodies, Negative $\overline{GM}$

What happen when the $\overline{GM}$ negative values or in other words the height (draft) increases beyond the critical height ratio? The body will flip into the side and turn to the next stable point (angle). This is not a hypothetical question, but rather practical. This stable point (that is body at new angle) will be point where body will be rotating around. At that point, the possible rotation swing to the deck is smaller and thus the ship is less stable.

This situation happens when a ship is overloaded with containers above the maximum height. In commercial ships, the fuel is stored at the bottom of the ship and thus the mass centroid (point G) is changing during the voyage. So, the ship that was stable (positive $\overline{GM}$) leaving the initial port might became unstable (negative $\overline{GM}$) before reaching the destination port. It is the responsibility of the officer (captain) to be aware of this pitfall.

9.3 The $\overline{GZ}$ diagram

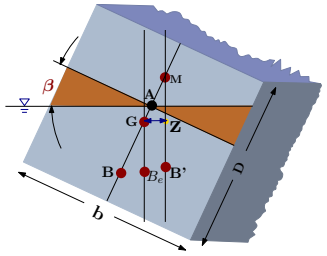


Fig. 9.6 – The right arm distance is display as $\overline{GZ}$ as a function of β

A typical approach in stability instead of looking at the value of $\overline{GM}$ a different imaginary distance is checked. This distance is what called the right hand arm. This refers to distance between the gravity and buoyancy forces. In Fig. 9.6 depicts the distance $\overline{GZ}$. This distance is perpendicular to the line $\mathcal{B}'\mathcal{M}$. The line $\mathcal{G}\mathcal{B}_e$ is parallel to $\mathcal{B}'\mathcal{M}$. $\mathcal{B}_e$ is the buoyancy point that obtained when the equilibrium is achieved. In other words, this point is the point the regular B' that travels through until the stability point. It can be noticed that his change occurs when the body is rotated and it no question stability. In this analysis, the question is the location of buoyancy centroid. This examination is based on the size of the right turning arm. When the arm length is zero the floating body is stable or that is point like watershed line after which the body will turn to next stable point.

In this analysis, the buoyancy centroid will be determined. The simple way to explain this point is by example. First body that will be dealt is the extruded rectangular like the one show in Fig. 9.6. Utilizing Eq. (4.20) provides the buoyancy centroid. For simplicity, first the vertical distance between the buoyancy centroid to the liquid of the water is calculated. Later, the horizontal distance between the gravity centroid and the buoyancy centroid.

The vertical distance can asses by finding first distance $\mathcal{B}'L$ which can ascertained by

$$\mathcal{B}'L = C_\xi - LN = C_\xi - C_\eta \tan\beta \tag{9.7}$$

The vertical distance is then

$$\overline{VD} = \mathcal{B}'L \cos\beta = (C_\xi - C_\eta \tan\beta)\cos\beta \tag{9.8}$$

Dividing equation by draft, D yields

$$\frac{\overline{VD}}{D} = \left(\frac{C_\xi}{D} - \frac{C_\eta}{D}\tan\beta\right)\cos\beta \tag{9.9}$$

Where C_ξ/D can be obtained from Eq. (4.21) plus utilizing Archimedes's formula as

$$\frac{C_\xi}{D} = \frac{\rho_r}{2} + \frac{b}{D}\frac{\tan\beta}{2} - \frac{\tan^2\beta}{24}\left(\frac{b}{D}\right)^2\frac{1}{\rho_r} \tag{9.10}$$

Where ρ_r is the density ratio of the solid body over the liquid. Utilizing Eq. (4.22d)

$$\frac{C_\eta}{D} = \frac{b}{2\,D} - \frac{1}{12}\left(\frac{b}{D}\right)^2\frac{\tan\beta}{\rho_r} \tag{9.11}$$

Before the results are presented the maximum angle β has to addressed. It can notice that the displaced volume is fix and independent of the angle of inclination. The maximum angle that body can turn before it get over the edge is

$$\tan\beta_{max} = \begin{cases} \frac{2\,d}{b} = \frac{2\,\rho_r}{\frac{b}{D}}, & \text{for } \rho_r \leq 0.5. \\ \frac{2\,(D-d)}{b} = \frac{2\,(1-\rho_r)}{\frac{b}{D}}, & \text{otherwise.} \end{cases} \tag{9.12}$$

The results are exhibited in Fig. 9.7. The maximum angle has a maximum at $\rho_r = 0.5$ with is results that the other corner is going to immersed in the liquid for a large value of density ratio. While, this point could be seem as trivial it is very important even with the calculations. At this point, the rate of change for moment of inertia obtains it minimum. This is a suspicious point in which it could be a stable point (e.g point where the body rotating around).

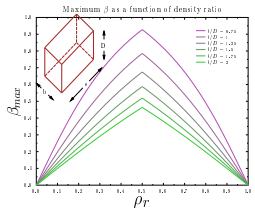
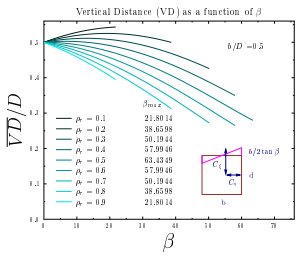
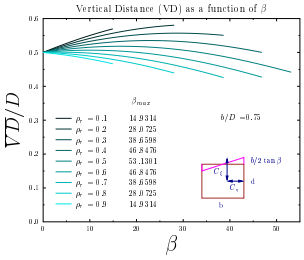


Fig. 9.7 – The maximum beta angle that before the corner is immersed in the liquid

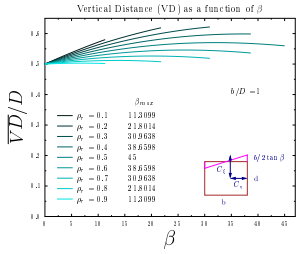
At this point the discussion can turn back to distances vertical and horizontal. The horizontal distance $\boldsymbol{GZ}$ has to go through several stages. First the vertical distance from the buoyancy (referred to as VD).



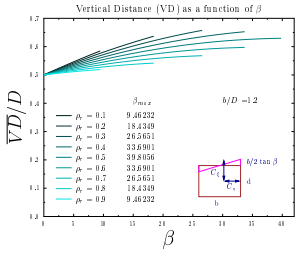
(a) Vertical distance for $b/D = 0.5$



(b) Vertical distance for $b/D = 0.75$



(c) Vertical distance for $b/D = 1.0$



(d) Vertical distance for $b/D = 1.2$

Fig. 9.9 – Vertical distance for rectangular extruded shape showing for 4 different b/D ratios as function of the turning angle β.

Fig. 9.9 shows the results of the vertical distance as a function for various density ratio and various b/D ratio. For small values value of b/D the vertical distance become smaller. For example, for $b/D = 0.5$ almost all the values are trending down. While the values treading up for large values see Fig. 9.8d. This trend is consistent with the Potential

Energy is based on. According to this method, the shorter distance of $\overline{BM}$ the more stable the situation is. Here, the stable situations will have a trend going up because they are stable and vice versa. The intermediate stage has a mix situation where part trending up and part trending down which indication that the body is stable only under certain conditions which is consistent with previous observations.

The distance $\overline{\boldsymbol{GZ}}$ is calculated utilizing the nomenclatures that used in Fig. 9.10. The distance $\overline{\boldsymbol{G}J}$ is obtained by

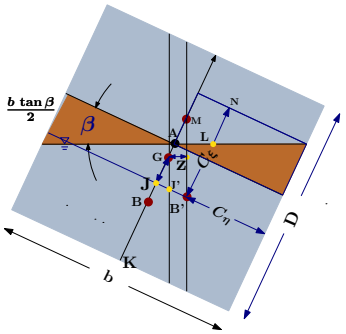


Fig. 9.10 – Schematic for naming for the $\overline{\boldsymbol{GZ}}$ for vertical and horizontal distances. Notice, some of the nomenclatures are adapted from page 46.

$$\overline{\boldsymbol{G}J} = d - \overline{JK} - \overline{\boldsymbol{AG}} \tag{9.13}$$

Where these distances are evaluated as $\overline{JK} = d + \Psi - C_\xi$ and $\overline{\boldsymbol{AG}} = d - D/2$. Utilizing these definitions yields

$$\boldsymbol{G}J = \frac{D}{2} + C_\xi - (\Psi + d) \tag{9.14}$$

The dimensionless value of C_ξ is given by Eq. (9.10). Utilizing the $\boldsymbol{G}J$ to obtain the

next value in the chain of values as

$$\overline{JJ'} = \overline{\boldsymbol{G}J} \tan\beta \tag{9.15}$$

with this value

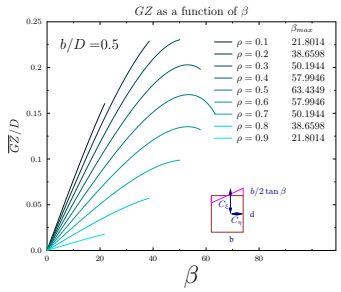
$$\overline{\boldsymbol{B}'J'} = \overline{JB'} - \overline{JJ'} \tag{9.16}$$

where the value of $\overline{JB'}$ can be observed from the diagram as

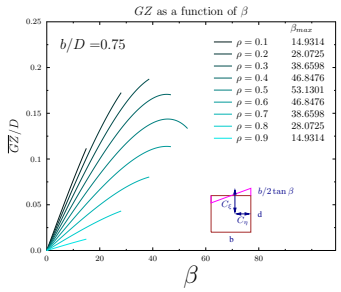
$$\overline{JB'} = \frac{b}{2} - C_\eta \tag{9.17}$$

and finally

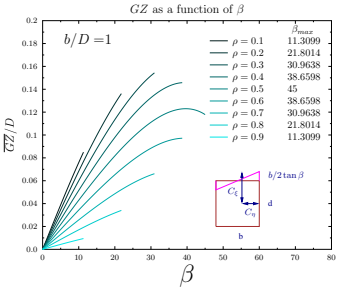
$$\boldsymbol{\mathcal{B}}'J'_\perp = \boldsymbol{GZ} = \boldsymbol{\mathcal{B}}'J' \cos\beta \tag{9.18}$$



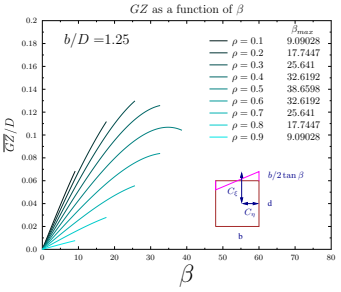
(a) $\boldsymbol{GZ}$ as a function β for b/D=0.5



(b) $\boldsymbol{GZ}$ as a function β for b/D=0.75



(c) $\boldsymbol{GZ}$ as a function β for b/D=1.0



(d) $\boldsymbol{GZ}$ as a function β for b/D=1.25

Fig. 9.12 – $\boldsymbol{GZ}$ as a function of the β for various b/D. It can be noticed that the angle is smaller for larger density ratio.

Several trends can be observed in Fig. 9.12. First it must be noted that of the abscissa scale is different for these graphs. As the ratio of b/D incenses the distance $\boldsymbol{GZ}$

decreases. The maximum is achieved earlier for larger b/D values. After the body reaches the maximum angle, new equation describes by the new equations and it is not shown in the figure. The change will manifest itself in the direction but still will be continues that is no jump in the angle.

9.4 Stability of Body with Shifting Mass Centroid

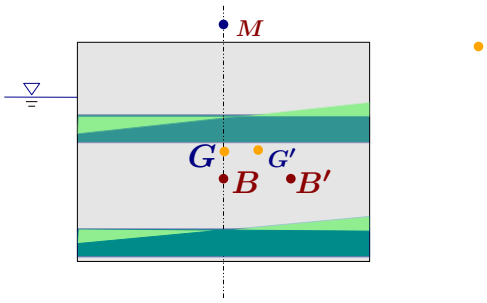


Fig. 9.13 – The effects of liquid movement on the $\overline{GM}$.

Ships and other floating bodies carry liquid or have a load which changes the mass location during tilting of the floating body. For example, a ship that carries wheat grains where the cargo is not properly secured to the ship. The movement of the load (grains, furniture, and/or liquid) does not occur in the same speed as the body itself or the displaced outside liquid. Sometimes, the slow reaction of the load, for stability analysis, is enough to be ignored. Exact analysis requires taking into account these shifting mass speeds. However, here, the extreme case where the load reacts in the same speed as the tilting of the ship/floating body is examined. For practical purposes, it is used as a limit for the stability analysis. There are situations where the real case approaches to this extreme. These situations involve liquid with a low viscosity (like water, alcohol) and ship with low natural frequency (later on the frequency of the ships). Moreover, in this analysis, the dynamics are ignored and only the statics is examined (see Figure 9.13).

A body is loaded with liquid "L" and is floating in a liquid "F" as shown in Figure 9.13. When the body is given a tilting position the body displaces the liquid on the outside. At the same time, the liquid inside is changing its mass centroid. The moment created by the inside displaced liquid is

$$M_{in} = g\,\rho_{l\,L}\theta I_{xx\,L} \tag{9.19}$$

Note that $I_{xx\,L}$ isn't the identical to the moment of inertia of the outside liquid interface.

The change in the mass centroid of the liquid "L" then is

$$\overline{G_1G_1'} = \frac{\cancel{g}\,\cancel{\rho_{l\,L}}\theta I_{xx\,L}}{\underbrace{\cancel{g}\,V_L\,\cancel{\rho_{l\,L}}}_{\substack{\text{Inside}\\ \text{liquid}\\ \text{weight}}}} = \frac{I_{xx}\,\theta_L}{V_L} \tag{9.20}$$

Equation (9.20) shows that $\overline{GG'}$ is only a function of the geometry. This quantity, $\overline{G_1G_1'}$, is similar for all liquid tanks on the floating body.

The total change of the vessel is then calculated similarly to center area calculations.

$$g\, m_{total}\, \overline{GG'} = \cancel{g\, m_{body}}^{0} + g\, m_f \overline{G_1G_1'} \tag{9.21}$$

For more than one tank, it can be written as

$$\overline{GG'} = \frac{g}{W_{total}} \sum_{i=1}^{n} \overline{G_iG_i} \rho_{l_i} V_i = \frac{g}{W_{total}} \sum_{i=1}^{n} \frac{I_{xxbi}}{V_{bi}} \tag{9.22}$$

A new point can be defined as G_c. This point is the intersection of the center line with the vertical line from $\boldsymbol{G'}$.

$$\overline{\boldsymbol{G}\, G_c} = \frac{\overline{GG'}}{\sin\theta} \tag{9.23}$$

The distance that was used before $\overline{\boldsymbol{GM}}$ is replaced by the criterion for stability by $\overline{\boldsymbol{G_c\, M}}$ and is expressed as

$$\overline{\boldsymbol{G_c\, M}} = \frac{g\, \rho_A\, I_{xx\,A}}{\rho_s V_{body}} - \overline{\boldsymbol{GB}} - \frac{1}{m_{total}} \frac{I_{xxb}}{V_b} \tag{9.24}$$

If there are more than one tank partially filled with liquid, the general formula is

$$\overline{\boldsymbol{G_c\, M}} = \frac{g\, \rho_A\, I_{xx\,A}}{\rho_s V_{body}} - \overline{\boldsymbol{GB}} - \frac{1}{m_{total}} \sum_{i=1}^{n} \frac{I_{xxbi}}{V_{bi}} \tag{9.25}$$

One way to reduce the effect of the moving mass center due to liquid is done by substituting a single tank with several tanks. The moment of inertial of the combine two tanks is smaller than the moment of inertial of a single tank. Increasing the number of tanks reduces the moment of inertia. The engineer could design the tanks in such a way that the moment of inertia is operationally changed. This control of the stability, $\overline{\boldsymbol{GM}}$, can be achieved by having some tanks spanning across the entire body with tanks spanning on parts of the body. Movement of the liquid (mostly the fuel and water) provides way to control the stability, $\boldsymbol{GM}$, of the ship.

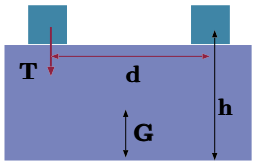


Fig. 9.14 – Measurement of GM of floating body.

9.4.1 Neutral frequency of Floating Bodies

Please ignore this section as it will completely be modified.

Meta

The examination of the $\boldsymbol{GZ}$ can be treated as linear for small range. Hence, this fact can be basically making the ship or the floating body act as it is a pendulum in some senses. Yet at the present case the mass or the moment of inertia is not easy to calculate. In movement of pendulum in air the added mass and the add moment of inertia are hard to calculated and is not negligible. The governing equation for the pendulum is

$$\ell\ddot{\theta} - g\,\theta = 0 \tag{9.26}$$

Where here ℓ is length of the rode (or the line/wire) connecting the mass with the rotation point. Thus, the frequency of pendulum is $\frac{1}{2\,\pi}\sqrt{\frac{g}{\ell}}$ which measured in Hz. The period of the cycle is $2\,\pi\,\sqrt{\ell/g}$. Similar situation exists in the case of floating bodies. The basic differential equation is used to balance and is

$$\overbrace{I\ddot{\theta}}^{rotation} - \overbrace{V\,\rho_s\,\overline{\boldsymbol{GM}}\,\theta}^{rotating\ moment} = 0 \tag{9.27}$$

In the same fashion the frequency of the floating body is

$$\frac{1}{2\,\pi}\sqrt{\frac{V\,\rho_s\,\overline{\boldsymbol{GM}}}{I_{body}}} \tag{9.28}$$

and the period time is

$$2\,\pi\sqrt{\frac{I_{body}}{V\,\rho_s\,\overline{\boldsymbol{GM}}}} \tag{9.29}$$

In general, the larger $\overline{\boldsymbol{GM}}$ the more stable the floating body is. Increase in $\overline{\boldsymbol{GM}}$ increases the frequency of the floating body. If the floating body is used to transport humans and/or other creatures or sensitive cargo it requires to reduce the $\overline{\boldsymbol{GM}}$ so that the traveling will be smoother.

Meta End

9.4.2 Limitations

This topic is dealing with several aspects of fluid mechanics like wave propagation on shallow liquid and the add mass. Admittedly these topics are very advance and require considerable amount of material to explain them. Simplified explanation is provided. The take away of this section to illustrate that the analysis is is provides the limit and the actual movement are some what different. When the ship or the floating body undergoes rolling,the shape of the body is fixed (to very large degree) while the shape or the configuration of the liquid in the container or the tank is not. The actual time to get

the shape that should be assumed to be in a steady state. Steady state refers to a state which obtained when the particles stay at that location for ever (again approximate definition). How long it takes for the liquid to reach a steady state is very hard to calculate. However, it (to ascertain) is a much easier to do. First, it has to define the characteristic time (the typical time to for liquid to move from side to side).

The movement of the liquid from side to side is done by a wave (this claim is without a proof and more about it will appear in the book of "Basic of Fluid Mechanics" by this author). The speed of the wave over liquid in a shallow liquid is approximated by gravity and the liquid depth.

$$U \sim \sqrt{h\,g} \tag{9.30}$$

This equation provides a reasonable estimate of the velocity. The time required is then

$$t_c = \frac{b/2}{\sqrt{h\,g}} \tag{9.31}$$

When writing Eq. (9.31), it was assumed that the average distance is about the half the width $(b/2)$ (liquid is moving side to side). Several additional assumptions have to be made to find the characteristic time liquid movement. What is the

When the body or the ship is rolling the

9.5 Metacentric Height, $\overline{\boldsymbol{GM}}$, Measurement

The metacentric height can be measured by finding the change in the angle when a weight is moved on the floating body.

Moving the weight, T a distance, d then the moment created is

$$M_{weight} = T\,d \tag{9.32}$$

This moment is balanced by

$$M_{righting} = W_{total}\overline{\boldsymbol{GM}}_{new}\,\theta \tag{9.33}$$

Where, W_{total}, is the total weight of the floating body including measuring weight. The angle, θ, is measured as the difference in the orientation of the floating body. The metacentric height is

$$\overline{\boldsymbol{GM}}_{new} = \frac{T\,d}{W_{total}\,\theta} \tag{9.34}$$

If the change in the $\overline{\boldsymbol{GM}}$ can be neglected, equation (9.34) provides the solution. The calculation of $\overline{\boldsymbol{GM}}$ can be improved by taking into account the effect of the measuring weight. The change in height of $\boldsymbol{G}$ is

$$\not{g}\,m_{total}\,G_{new} = \not{g}\,m_{ship}\,G_{actual} + \not{g}\,T\,h \tag{9.35}$$

Combining equation (9.35) with equation (9.34) results in

$$\overline{\boldsymbol{GM}}_{a}ctual = \overline{\boldsymbol{GM}}_{new}\,\frac{m_{total}}{m_{ship}} - h\,\frac{T}{m_{ship}} \tag{9.36}$$

The weight of the ship is obtained from looking at the ship depth.

9.6 Stability of Submerged Bodies

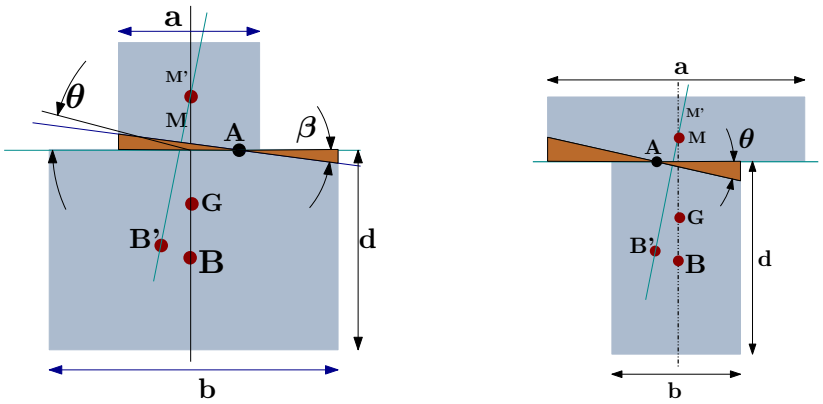
The analysis of submerged bodied is different from the stability when the body lays between two fluid layers with different density. When the body is submerged in a single fluid layer, then none of the changes of buoyant centroid occurs. Thus, the mass centroid must be below than buoyant centroid in order to have stable condition. Stable conditions

However, all fluids have density varied in some degree. In cases where the density changes significantly, it must be taken into account. For an example of such a case is an object floating in a solar pond where the upper layer is made of water with lower salinity than the bottom layer(change up to 20% of the density). When the floating object is immersed into two layers, the stability analysis must take into account the changes of the displaced liquids of the two liquid layers. The calculations for such cases are a bit more complicated but based on the similar principles. Generally, this density change helps to increase the stability of the floating bodies. This analysis is out of the scope of this book (for now).

9.7 Stability of None Systematical or "Strange" Bodies

This topic was address somewhat in the previous chapter and will be expanded in the following versions of this book. Previously, the discussion dealt with smooth transition in which the cross section area is a smooth function of the height. In section the discussion deals with abrupt change in the cross section. The change is more profound. Consider two extreme cases, one where narrow immersed body and two the wide part immersed. The rotation point A is not rotating and move in two different directions depending on cross section area (as function of the height) direction. Clearly without calculating any data, one can observed that the body with the wide immersed part rotating point A moves to the right while the other case the rotating point A moves to left. In, the change if direct of the rotating point change the way calculations are carried out. Never the lest it is clear wide immersed body is more stable.

While most floating bodies are symmetrical or semi–symmetrical, there are situations where the body has a "strange" and/or un–symmetrical body. The calculations will appear in the following versions. These are on the same considerations that that were discussed in the previous chapter.



(a) Floating Body with abrupt transition to small on the top

(b) Floating Body with abrupt transition to large on the top

Fig. 9.15 – Calculations of $\overline{GM}$ for abrupt shape body.

10

Variable Density Floating Bodies

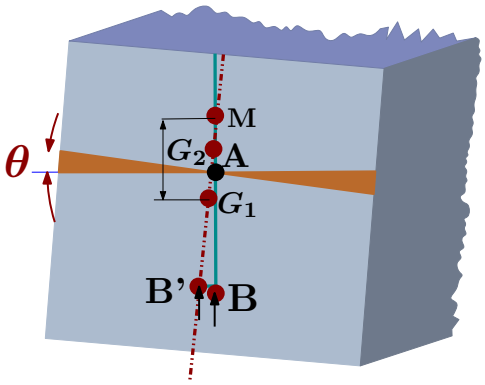


Fig. 10.1 – Body with movable gravity centroid.

Ship and other floating bodies do not have a constant density. In other words, the gravity centroid moves depend on the loading to a different location in the ship or body. That point is different from the centroid of the entire body. However, the buoyancy centroid is in the same location. located before as it depend on the displaced volume. If the gravity centroid is below the buoyancy centroid then the body is stable and no further calculations are needed. However, for most cases the issue to find how much to change the location of the gravity centroid to the vessel is stable. This chapter is dealing with the requirement on location of the gravity centroid for given geometry.

Previously the gravity location was given for the question was given a rotation of the body by θ what is the resulting α. Now the question is more complicated. Given

the geometry, what are the requirements on the location of the gravity centroid to make the body stable. A new triangle can be build that is now made from tree points B, B' and G.

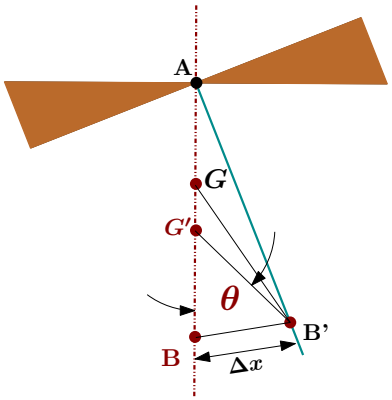


Fig. 10.2 – Diagram to explain the calculation for safe $\boldsymbol{G}$.

The geometry (law of sines) it can be written and utilizing Eq. (4.15)

$$\frac{\Delta x}{\sin\theta} = \frac{\mathcal{B}\boldsymbol{G}'}{\sin 90^\circ} \longrightarrow \frac{\cancel{\tan\theta}\dfrac{I_{xx}}{V_0}}{\cancel{\sin\theta}} = \mathcal{B}\boldsymbol{G}' \tag{10.1}$$

Or in a clear form

$$\mathcal{B}\boldsymbol{G}' = \frac{I_{xx}}{V_0} \tag{10.2}$$

The location of gravity in solid body (or floating body) is at $\boldsymbol{G}$ and normally is at the centroid of the entire body. The distance required to lower the gravity to make the body stable is $\boldsymbol{GG}'$. Hence,

$$\boldsymbol{GG}' = \mathcal{B}\boldsymbol{G} - \mathcal{B}\boldsymbol{G}' \longrightarrow \boldsymbol{GG}' = \mathcal{B}\boldsymbol{G} - \frac{I_{xx}}{V_0} \tag{10.3}$$

After the required distance is calculated, a weight needed to be attached, to bottom, or other place. The weight size is based on the calculations explained in Chapter 4.

Example 10.1: Missing GM for Ship — **Level: Easy**

In Ex. 8.2 dealt with rectangular shape which has zone with were unstable. Here in this example, a possible correction will be calculated ($\boldsymbol{GG}'$).

Solution

The Fig. 8.13 shows the shape where the rectangular is unstable and for that range

End of Ex. 10.1

the required change is calculated. Eq. (8.2.e) is copied here for the boundary.

$$\frac{b}{D} = \sqrt{6\,\frac{\rho_s}{\rho_\ell}\left(1-\frac{\rho_s}{\rho_\ell}\right)} \tag{10..a}$$

and

$$\frac{d}{D} = \frac{\rho_s}{\rho_\ell} \tag{10..b}$$

In this case $\mathcal{B}\boldsymbol{G} = D/2 - d/2$ and thus the $\boldsymbol{GG'}$ is

$$\boldsymbol{GG'} = \left(\frac{D}{2} - \frac{d\,D}{2\,D}\right) - \frac{1}{12}\,\frac{b^2\,D}{D^2}\,\frac{\rho_\ell}{\rho_s} \tag{10..c}$$

and can be combined

$$\frac{\boldsymbol{GG'}}{D} = \frac{1}{2}\left(1-\frac{\rho_s}{\rho_\ell}\right) - \frac{1}{12}\left(\frac{b}{D}\right)^2\,\frac{\rho_\ell}{\rho_s} \tag{10..d}$$

It can be observed that the largest change should be made when the density is when the liquid density is relatively very law. Most large ships are with density ratio on the upper limit has the change is relatively small.

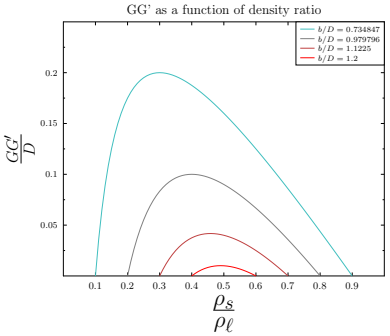


Fig. 10.3 – Requirements on gravity centroid for rectangular shape body.

The next is similar for the cylinder that was dealt before.

Example 10.2: What cylinder is stable **Level: Hard**

Earlier in the book in Fig. 8.24 the stability of cylinder was discussed. Here, the correction that need to be applied so that cylinder will be stable ($\boldsymbol{GG'}$).

Solution

The Fig. 8.13 shows the shape where the rectangular is unstable and for that range

End of Ex. 10.2

the required change is calculated. stability zone.

$$\frac{r}{D} = \sqrt{2\,\frac{\rho_s}{\rho_\ell}\left(1-\frac{\rho_s}{\rho_\ell}\right)} \tag{10..e}$$

and

$$\frac{d}{D} = \frac{\rho_s}{\rho_\ell} \tag{10..f}$$

In this case $\mathcal{B}\boldsymbol{G} = D/2 - d/2$ and thus the $\boldsymbol{GG'}$ is

$$\boldsymbol{GG'} = \left(\frac{D}{2} - \frac{d\,D}{2\,D}\right) - \frac{1}{4}\,\frac{r^2\,D}{D^2}\,\frac{\rho_\ell}{\rho_s} \tag{10..g}$$

and can be combined

$$\frac{\boldsymbol{GG'}}{D} = \frac{1}{2}\left(1-\frac{\rho_s}{\rho_\ell}\right) - \frac{1}{4}\left(\frac{r}{D}\right)^2\,\frac{\rho_\ell}{\rho_s} \tag{10..h}$$

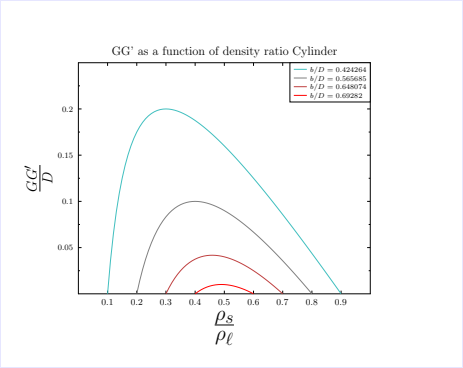


Fig. 10.4 – Requirements on gravity centroid for cylinder shape body.

The Fig. 10.4 is seem to the replica (hopefully) not a mistake.

Example 10.3: EverGiving $\mathcal{GM}$

Level: Easy

The famous ship Ever Giving for this excise can be consider as extruded square box with the following dimension L $= 400[m]$ b $= 60[m]$ and the D $= 60[m]$. The maximum draft (the depth) the ship can be in the liquid is $16[m]$. What is the minimum requirement on gravity centroid.

Solution

The ratio of $b/D = 1$ in this case. From the value of d and D the ratio of density can be estimated as $16/60 = 0.267$ and observing from Fig. 10.3 the value of $\boldsymbol{GG'}/D = 0.075$ thus, the value of $\boldsymbol{GG'} = 60[m]\times 0.075 \sim 4.5[m]$ For uniform body the gravity centroid

End of Ex. 10.3

has to be at half of D thus it is has to be at $30[m]$. Thus, subtracting the correction it became that gravity centroid has to be at least below the $25[m]$ from the bottom of the ship.

11 Rolling

11.1 Introduction

While ships are build to be stable (for the most part), there are several situations where it is not sufficient. Ships act as a giant pendulum but with several distinct characteristics (varying hinge point, varying moment of inertia, changing the damping elements etc) which will be discussed in this chapter. Under certain conditions, these giant pendulums create a problem for the cargo on board. This issue leads to accidents in which cargo (containers) damaged. This problem could to be due to the resonance of the ship with cargo (there could other reasons). This chapter focus on this issue. There numerous investigations with estimate cost of over 10 millions dollars while cost of the actual damages was high as a billion dollars. Thus, this issue deserves a chapter examining the various factors of this problem and mechanisms in which they created. In this chapter, a discussion is presented on the general parameters and a dimensional analysis of the problem.

11.1.1 The Quasi–State Foundation

It is essential to realized that quasi–state is necessity in order to be able to analysis this complicate situation. To make this point, an analogy to thermodynamics is provided. To understand thermodynamics, the material is categorized into equilibrium and none–equilibrium zones. That is, initially classes use and presented the steam tables and treated as Holy Grail for example, water boils at about 100°C at atmospheric pressure. This ability to carry calculations, based on this rough estimate, simplifies the ability to obtain reasonable results. Yet, when precise values are required the effect of the heat

flow rate, surface tension, and boiling surface must be taken into account[1]. The real reason for this separation is created helpful system so engineers simplify the calculations. The same should be done for stability, and the quasi–equilibrium study should be done before the full dynamics is implemented.

The point where the rotation (vibrations) occurs is important to established in any pendulum. Thus, the movement in z, known as heave, changes the location of the rotation point. A change in x and y also causes the change in the rotation point but to a smaller extent. In the analysis, these changes should be ignored and quasi–state is assumed. Thus, in this chapter when the rotation point is needed for calculation, this assumption is utilized.

Obviously, the location of the rotation hinge of the pendulum is important to the ship movement. The value of the moment of inertia depends on this location. Yet, numerous works deal with the dynamics situations (where the ship is not in equilibrium in z direction), yet, the rotation hinge is not known. In dynamics situations the hinge point deviates slightly from the quasi–equilibrium. Hence, the knowledge this hinge point at quasi–equilibrium is essential and should be known before tackling the dynamics aspects. In this chapter, deals with quasi–equilibrium state and attempt to establish it. Furthermore, the deviations from the normal such as trim and list are not dealt at this stage.

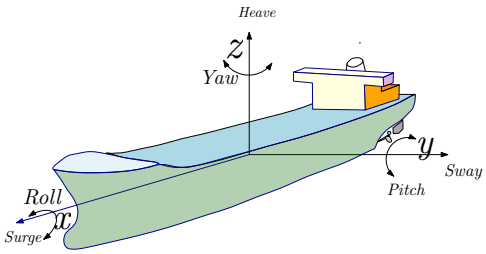


Fig. 11.1 – Six motions of ship with their names (Floating bodies).

It to be established here that there is **no** problem with the linear motions: heave, sway, surge.The sway and surge (check Fig. 11.1), no ship will be capsizing because these motions (neglecting the transfer of energy mechanisms)[2]. In the extreme the heave, in theory, can cause drowning of the floating body if the magnitude of movement is such it exposed the opening to the liquid. However, if there is enough extra floating, most waves amplitude are not sufficient to capsizing which are not common (this author is not aware of a single case of such issue was for a commercial ship)[3] .

[1]The purity of the water is deliberately does not appear in the list because they common and yet less important. For example, the change in salinity for common material like sea water can be a few degree while the smooth surface change the boiling temperature by about 30°C or more.

[2]This remark should be limited by suggestion offered by several individuals that tsunami can turn even large ship. Yet, this remark is true for "normal" waves.

[3]The author was supposed to be served on Mazada owned by Zim Israel Navigation Co. of Haifa on her last vogue 1981. The ship sank because admission of to much water due to leaking storage compartments. But, the ship did not because of heave movement.

Example 11.1: Heave motion **Level: Intermediate**

A large container ship (the Ever Giving) with the size of 60 [m] width and 400 [m] long is pushed 1[m] into the sea. Estimate the force that requires to create this movement. How much water the ship has to be covered to create this force. The ship height is 60 [m] and ship is about 50% inside the water. How large force is required to be to push this ship 25% down? What is the estimated acceleration of ship at this point?

Solution

The weight of the 1 cubic meter of water is about 1 tonne which for simplicity, this value is taken as standard for this example. The force is then simply evaluated as $60\times400 = 24,000\ tonnes$. To push the ship 1 [m] the ship has to be covered with 1[m] of water. Large ship do not covered with a miters of water and thus heave movement is limited. To have 25% movement it has to be covered by the same amount of water. In case of Ever Giving the entire ship has to be covered by 15 [m] of water (it is very fast wave of 45 [m] and very long 400 [m] long). The force is $15 \times 60 \times 400 tonnes$. The question of the acceleration is important because it determines the time the body will be under the liquid which determines possibility of drowning. It can be shown that this situation (to be constructed example in chapter 12) it analogous to a block with density of half of the liquid at depth of 15[m]. The mass of the block is $V\,\rho_s$ and the net force is $V\,(\rho_\ell - \rho_s)$. The acceleration is the ratio of these two quantities

$$a = \frac{V\,(\rho_\ell - \rho_s)\,g}{V\,\rho_s} = \left(\frac{\rho_\ell}{\rho_s} - 1\right)\,g \tag{11.1.a}$$

which in this case is about g. Thus, to cause drowning the wave speed has to be in order magnitude or larger than $\sqrt{\frac{2\,L}{g}}$. Where L in this case is the ship length.

It was demonstrated that the heave is not dangerous up to the point where the ship openings is exposed to the liquid. The heave movement is mainly concern of the increase of the resistance to the flow (ship movement). The yaw rotation (again without the transfer mechanism) is not dangerous because the obvious observation (see Fig. 11.1). The pitch rotation can be caused by either by the transfer mechanisms or waves from the x or y (non symmetrical wave) directions. Of the three rotation motions the rolling is the most important and dangerous to the ship. It not to say that there are no situations where a small boat along the pitch rotation can overturned (the author personally observed such case of capsizing). These situations simply are less common and specialty case study. After examining numerous papers on the effects of rolling this author had a strong urge to utilize the standards fluid mechanics tools to write a governing equation(s). As usual, the examination previous work will be followed by a discussion on various terms of the governing equation with circumstances under which the equation can be simplified. Due to the mathematical difficulties, the previous work focus was on the mathematical aspects of the problems. That is, little care was given to the physical meaning of the terms and mainly focus was on solving the mathematics. For example, (Taylan 2004; Taylan 2000) considered the speed of ship and damping

force as the mathematical functionality.

The rolling rotation has unique features that makes it very difficult to understand as compared to regular pendulum rotation motion. The fact is that until this point, no one even formulate the complete governing equation[4] and discuss what physical elements and how they interact. The "regular" rotation has forces/moments balanced around a fix point which may or may not have external inputs (sometimes referred as exciting input). Floating body among other things has several elements that are unique such shifting rotation point and shifting mass and/moment of inertia. The second issue is transformation mechanism from one rotation to another which will be addressed in this chapter. This author has not found a single source to which discuss the transfer mechanism from one rotation mode to another with the exception of the wave in the x direction or surge that results in a rolling motion (that is in other words heave and rolling). The basic theory for this effect is that a wave moves along the ship change $\boldsymbol{GM}$ during the time ship passes the wave. This idea as far it can be judged was suggested by France and Shin (France, Levadou, Treakle, Paulling, Michel, and Moore 2003; Shin, Belenky, Paulling, Weems, and Lin 2004).The ship, according to this theory, has to be exposed to a waves with a frequency of twice the natural ship frequency (van Laarhoven 2009) which is defined as

$$\omega_\phi = \sqrt{\frac{\rho_\ell \, g \, V_0 \, GM}{I_{xx} + M_{add}}} \tag{11.1}$$

where ω_ϕ is the natural frequency and M_{add} is the added mass of the ship[5] and the other properties are regular terms used in the book This equation can be recognized as the regular term from a simple pendulum. In addition, the wave length has to be at certain conditions (while they are not specified). In the said paper, it seem from the equation has a simple units mistake and the added mass should be replaced by the added moment of inertia (logically and dimensionally). (to be added as example to the dimensional analysis chapter) It is not clear from the model what causes the rolling beside that the rolling is possible if the natural frequency was correct. This model disregards the ship shape, shifting rotation point, and other important characteristics. There are several problems with the assumptions that this model build on and they will be discussed later.

When engineers do not understand a certain topic, a committee (unknown 2008; Krata and Wawrzyński 2016) issue recommendation with total disregard to the physics. For example, IMO issue in 2008 equation for estimate the frequency (time of the period)[6] as

$$\tau = \frac{2\,c\,b}{\boldsymbol{GM}_0} \tag{11.2}$$

[4]Several individuals wrote to dispute this point. Yet when they were asked about how they calculate the ship moment of inertia and around what point the ship is rotated around, why were clueless. This point is correct regardless to individuals feelings. Yet, it demonstrate how entrenched fix rotation point concept etc, individuals even not aware how wrong they are!

[5]Should be moment of the inertia

[6]One can only wonder why the 2 is swallowed into coefficient c.

where[7]

$$c = 0.373 + 0.023\frac{b}{D} - 0.00043\,s \tag{11.3}$$

where:
c is coefficient describing ships transverse gyration radius,
b ship width, s ships length at waterline,
D denotes the ships draft, and
$\boldsymbol{G\mathfrak{M}}_0$ initial transverse metacentric height.
It is interesting in this equation that equation (committee) recognized that the compartmental approach is erroneous. Yet, the physical sources of the energy transfer from one dimension to another approach (or to multi–dimensional movements) are magical. Clearly this equation is only valid only in special cases at this stage to be considered unknown.[8]

11.2 Historical Background

One of the latest work (Remola 2018) formulated the governing equation as

$$I_{xx}\,\frac{d^2\theta}{dt^2} + M_D\,\left(\theta, \frac{d\,\theta}{dt}\right) + M_R\theta = M_W(t) \tag{11.4}$$

where M_D is the damping function which is depend on the angle and the angular velocity, $M_R \cong \boldsymbol{GZ}\,\rho_\ell\,g\,V_0$ (where the arm is $\boldsymbol{GZ}$ and $\rho_\ell\,g\,V_0$ is weight). The (mass) moment of inertia, I_{xx}, is in the yz plane and is assumed to be calculated for the rotation around $\mathfrak{M}$ point (metacenter and it is assumed to be fixed and include the added moment of inertia. No research was reported at what the angle for the added moment of inertia moment of inertia was calculated. In fact there are researches papers pointing that the added moment of inertia of circular shape (or very close to it) has some value while theoretically it must be zero (maybe it was rotated around another point. Anyone who knows how this happened?). Various sources are assuming that the causing the rolling is the waves from different directions. In construction, this equation it was assumed no coupling with other motions in the other directions based on (Spyrou and Thompson 2000). Some other accept to this assumption like (Bulian, Francescutto, and Fucile 2009) which are based on "because a good balance is achieved between simplicity and accuracy."

The actual equation that most researchers are trying to solve is without the damping effect. An example of such work on the ship rolling is by (Jovanoski and Robinson 2009) pointing to quite well phenomena to stability zones that dampingless equation created. Under the assumption of specific wave it is assumed (Allievi and

[7]The author would recommend that these kind of equations should be in dimensional analysis section. Keep in mind when the various coefficients given their dimensions also should be provided.

[8]This author speculated that about the affecting parameters. Yet, it will appear in later version.

Soudack 1990) that the governing can be written as

$$\frac{d^2\theta}{dt} + \overbrace{\frac{M_R}{I_{xx}}}^{{\omega_0}^2}\theta = W_A\,\cos(\omega_W\,t) \tag{11.5}$$

where ω_0 natural frequency ω_W disturbing wave W_A is the normalized exciting amplitude.

Example 11.2: The Natural Frequency **Level: Intermediate**

This question is inspired by the book (Rawson and Tupper 2001) it depicted here to demonstrate how much the field is frozen in time.

"A box with a length of 50[m], width 12[m], draught 3[m] above the water (freeboard) 6[m]. The height of the gravity centroid above the bottom is 4.5[m]. Assuming that the weight is uniformly distributed throughout the length and section of the vessel, and neglecting the effects of associated water, calculate the free periods of roll, pitch and heave in salt water."
Converting removing some of jargon from the question. It also had to added the hidden assumptions like neglect the added moment of inertia, resistance liquid etc.

Solution

The gravity centroid is given in the data and it is excessive information. The ship length should be used for pitch frequency. The buoyancy centroid is given by $3[m]/2 = 1.5[m]$. The value of $\boldsymbol{\mathcal{GB}} = 4.5 - 1.5 = 3[m]$. The value of $\boldsymbol{\mathcal{BM}}$ can be calculated as $I_{xx}/V_0 = (I_{xx}/s)/(A_0)$. The moment of inertia is

$$\frac{I_{xx}}{s} = \frac{b^3}{12} = \frac{12^3}{12} = 144[m^3] \tag{11.2.a}$$

The area is

$$A_0 = b\,d = 12 \times 3 = 36[m^2] \tag{11.2.b}$$

Thus $\boldsymbol{\mathcal{GM}}$ is $144/36 - 3 = 1[m]$. Notice that using the table will provide the same result. The natural frequency is

$$\omega = \sqrt{\frac{M_R}{I_{xx}}} \tag{11.2.c}$$

It must be noticed that even though the notation of I_{xx} for both cases the moment of inertia is referring to the mass (not m but I) rotating around line for box is rotating. Moment of inertia box (cuboid) around it center line (that is a line parallel to x the goes center of the box is

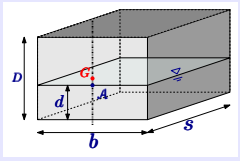


Fig. 11.2 – Cuboid For frequency analysis notice the change of moment of inertia.

continue Ex. 11.2

$$I_{xx} = \frac{1}{12}\overbrace{\rho_\ell\, d\, b\, s}^{box\ mass}\ g\ \left(D^2 + b^2\right) \tag{11.2.d}$$

The actual rotation will be around point $\boldsymbol{\mathcal{A}}$ which is at the liquid surface. In this case, the density is such the change moment of inertia is $\boldsymbol{G\mathcal{A}} = 4.5 - 3 = 1.5[m]$ and the change value to be $\Delta I_{xx} = 1.5^2\, mass$. For comparison the ratio between and I and ΔI is

$$\frac{\Delta I_{xx}}{I_{xx}} = \frac{\cancel{m}\boldsymbol{G\mathcal{A}}^2}{\frac{\cancel{m}}{12}\left(b^2 + D^2\right)} = \frac{12\cancel{D^2}\left(\frac{1}{2} - \frac{\rho_s}{\rho_\ell}\right)}{\cancel{D^2}\left(\left(\frac{b}{D}\right)^2 + 1\right)} = \frac{12\left(\frac{1}{2} - \frac{\rho_s}{\rho_\ell}\right)}{\left(\frac{b}{D}\right)^2 + 1} \tag{11.6}$$

The results depicted in Fig. 11.3. It clear from the figure that the common approach to assume that the density ratio is insignificant is contain large and significant error of factor $2 - 4$ (hundreds percents). This error is less significant in very small range of $\rho_s/\rho_\ell \sim 0.5$

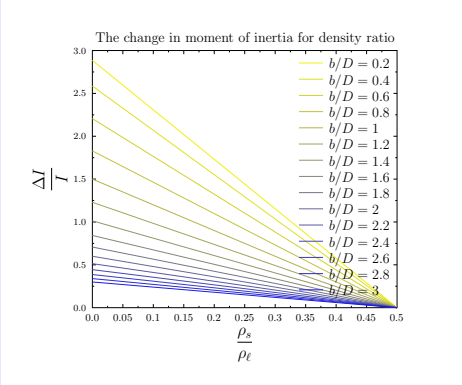


Fig. 11.3 – Effect the density ratio on the moment of inertia. Notice that presentation is in the range of 0–0.5 density ratio and the range of 0.5 to 1 should be symmetrical and thus it does not appear in the figure.

Clearly, the original author of the question was not aware of this effect. Even if the added mass and other effects were ignored the error due to the change of moment of inertia change the results. The density ratio is $3/(3+6) = 0.33$ and $b/D = 12/(3+6) = 1.3$. In this case, the error from the graph is $\Delta II \sim 0.75$. Or more precisely

$$\frac{12 \times (0.5 - 0.33)}{1.3^2 + 1} = 0.758$$

The weight of the ship can be ascertain by $\rho\, g\, d\, s\, b$ and it is $1.020 \times 9.8 \times 3 \times 50 \times 12 = 17992.8[kN]$ The moment of inertia of the cuboid is

$$I_c = \frac{1 \times 9.8 \times 3 \times 12 \times 50(9^2 + 12^2)}{12} = 330,750[tonne\, m]$$

End of Ex. 11.2

where I_c in this case is the moment of inertia around the center of the cuboid. The actual value around the rotation point is $330,750 * 1.758 = 581,458.5[tonne\, m]$ Another way to calculate $\boldsymbol{GM}$ is obtained by using Eq. (9.1.c) which is copied to here as well[a]

$$\boldsymbol{GM} = \frac{b^2}{12\,d} - \left(\frac{D}{2} - \frac{d}{2}\right) = \frac{12^2}{12 \times 3} - \frac{9}{2} - \frac{3}{2} = 1[m]$$

Now the calculation of the frequency.

$$\frac{M_R}{I} = \frac{\cancel{V_0 \rho_\ell}\, g \left[\frac{b^2}{12\,d} - \left(\frac{D}{2} - \frac{d}{2}\right)\right]}{\frac{1}{12}\cancel{V_0 \rho_\ell}(D^2 + b^2)} = \sqrt{g\, f(geometry)} \tag{11.2.e}$$

$$\omega = \sqrt{\frac{1 \times 9.}{a}}$$

[a]When discussing this question with a colleague she point out that the author is bit hypocrite not using dimensionless form. The undersign has to agree with her but he has to cater to his audience.

$$I_{xx}\, \frac{d^2\theta}{dt} + \left({\omega_0}^2 - W_A\, \cos(\omega_W\, t)\right)\, \theta = 0 \tag{11.7}$$

ω_W is natural frequency to the longitudinal wave frequency, and ω_0 is the natural frequency of the ship W_A is a measure of the waves amplitude. This equation is referred in the literature is Mathieu equation.

11.3 Physical Considerations

11.3.1 Added Mass and Added Moment of Inertia

At the current stage, it is important to discuss the physics of the situation. During the rotation of the ship beside the moment of inertia of the ship (around a point which will be discuss later) there additional material that is affected the ship moment of inertia. This material has a moment of inertia (around the same point) and is referred as the added moment of inertia. The added moment of inertia term has several issues that have to be addressed. This term appears in the current researchers derivation as a constant value. That is, the total moment of inertia expressed as $I_{ship} + I_{add}$ when both are considered to be constant (Remola 2018). Where add is denoting the addedAs Landweber and Miloh (Landweber and Miloh 1980) have shown that the added moment of inertia is constant for constant acceleration for the same geometrical

property shape. The added properties do not get negative values.[9]The added mass is always positive (none zero value). The added moment of inertia is zero or larger. The added moment of inertia of circle is zero when the rotation around the center. This fact should be a test point of any model on the added moment of inertia.

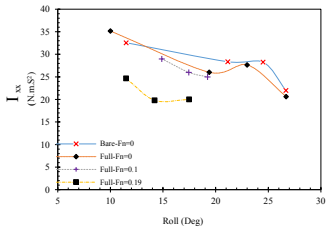


Fig. 11.4 – Added moment of inertia After Kianejad et al. Notice that the left subtitle was change from original to the standard in the book.

The second issue that affects the variation of the added moment of inertia is the change of the immersed geometry. For example, a square body starts as rectangular extruded area (depends on the density ratio) and changes to extruded trapezoid or extruded triangle depends on the angle). Hence, at every angle the body has different immersed geometry and therefore different added moment of inertia. The calculated Fig. 11.4 by Kianejad et al (Kianejad, Enshaei, and Ranmuthugala 2017) shows more or less constant value. If one accept this calculations that a constant value can be used[10]. If one insist on the constant value (reduction of accuracy idea) a better representation is in nature of[11] the added moment of inertia.

$$\overline{I_{add}} = 0.5\ (I_{add}|_{min} + I_{add}|_{max}) \tag{11.8}$$

Where $I_{add}|_{min}$ is the minimum added moment of inertia and $I_{add}|_{max}$ is estimate of the maximum In general this term should be written as

$$I_{add} = I_{add}|_0 + f(\theta) \tag{11.9}$$

The value of $I_{add}|_0$ is at zero angle and $f(\theta)$ can be positive or negative to maximum value of $I_{add}|_0$. Kianejad et al calculated the value of the specific ship of the added moment of inertia (Kianejad, Enshaei, and Ranmuthugala 2017). The authors did not specified the rotation point and used almost the full Nervier–Stokes equation (including the turbulence and shear stress.)[12]. The work by Kianejad et al was based on work by Korotkin and others (Korotkin 2008) which original definition of the added mass is based the surrounding material for ideal flow (that is the viscosity and turbulence

[10]This author did work on the specific code that done by researchers. However, one cannot wonder if the equations that were used are correct as they include viscosity and turbulence none which should be used in added properties. Thus, it possible that added moment of inertia is defined differently than the common added moment of inertia. The paper too abstract and does not specify what is really going on and hence it hard to ascertain the validity of the work. Several other papers by other research groups suggest this approach and yet none of them even provide the location of the rotation point for which it was calculated. In this stage, this undersigned takes this work to mean none standard added moment of inertia which some how include the dissipation terms.

[11]The source of the added moment of inertia is dragging of liquid due to the body geometry. Perfect cylindrical shape do no drag any mass on rotation because the fiction is not considered and thus the added moment of inertia has no added moment of inertia (yet it has added mass).

[12]The researchers could not answer the questions this author present to them.

must be ignored as these not part of the added properties.). Between these two works there were additional steps or works that and the research groups who was using these techniques. At writing this section it is not clear who introduce this strange definition of the added moment of inertia into "research" stream. The fact that there the added moment of inertia includes additional effects makes it none standard added properties. Due to the fact that these values are not based on known standards they should be ignored and hope that they should recalculated and converted to regular standard in the future.

Perhaps the biggest issue in understanding rolling is the change of location of the rotation point or hinge. As opposed to commonly believed, the rotation point is not at the metacenter but at the surface of the liquid (for quasi–equilibrium calculations and extruded liked bodies). For two–dimensional shape, it is at the middle of the liquid line.

Example 11.3: The Rotation location **Level: Advanced**

One misunderstood point in rotation of ship especially for rolling motion is the center of rotation. This point has profound effect on the frequency and other parameters. The liquid surface (wetted area) changes during the rolling motion. Demonstrate that the rotation point is the middle of the liquid surface for two dimensional shape.

Solution

As first approximation the ship is assumed to have a constant mass. This assumption is reasonable for the case where the ship shape does not have a large abrupt shape and the acceleration of the movement is relatively small. For practical purposes, a change of the mass up to 5% is still within zone for this analysis (much more complicate analysis results from stringent requirements).
When the ship mass is constant (quasi steady state) then the instantaneous displacement (assuming the heave movement is insignificant) the displacement volume is constant under these two assumptions. If the ship has continuous liquid surface (that is, the ship is not catamaran or trimaran) the rotation is at some point on the liquid surface. As long the ship shape is continuous, the subtracted area has to be equal added area. These two areas are, at the limit, strait triangles. The two triangles have the same area if the height and the base are the same in the two triangles. This condition occurs when the heights is the same which mean the rotation must occur at the middle the liquid surface.

The rotation point moves to a new location when the body/ship is turning to new angle because the middle of the liquid surface is at new location which depends on the geometry. Almost all the shapes of the ships are such that create a change in the hinge line or hinge point. The only exception to this perfectly extruded bodies. A cylinder or cuboid are an example of extruded body were the rotation location does not change. This effect has to be taken into account when calculating moment of inertia of body etc. The significance of this effect can be demonstrated by triangle up versus triangle down (which sometime stable in one way but not other) . Thus even the experimental attempts to measure it when ignoring the shifting of the hinge line (Balcer

2004) are not fruitful. Note, even the correct initial hinge line was erroneously identified at the metacenter (with the exception of on individual Lautrup (Lautrup 2011)[13]. The literature survey shows that not much work was done on these topics. For example Markeev (Markeev 2017) study a moving mass on the pendulum in 2017. Some what related work by Legaza (Legeza 2020) dealt with movable hinge for building. While some work has been done on this two topics, they not well understood and the theatrical base is still missing.

Example 11.4: Square Rotation Point Movement **Level: Intermediate**

A square is rotating 5°. Assume that density ratio is $\rho_s/\rho_\ell = 0.25$ Where is the location of the rotation point at that angle? How much the moment of the inertia change by percent?

Solution

The geometry is depicted in Fig. 11.5. The trapezoid area has to the same as before because the displaced volume (area) remain the same. The angle is given and according the above requirement the center must stay at the same physical location and hence the two triangles have the same area and cancel each other. Note the actual physical point moves (rotates). The body is rotating around the middle of the liquid surface (of the body).

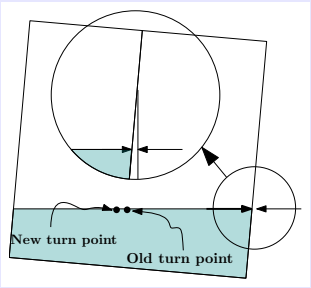


Fig. 11.5 – Change of the rotation point geometry. For this geometry these points are the same.

Example 11.5: Triangle Rotation Point Movemer **Level: Intermediate**

A triangle with the tip in side the liquid is turn about5°. Calculate the physical change of the rotation point. Assume that the triangle is isosceles with angle $2\,\alpha$ and the density ratio is 0.25 with side length a.

Solution

[13]It fact what missed by this author because later Lautrup made contradicting statement which confused this undersign.

continue Ex. 11.5

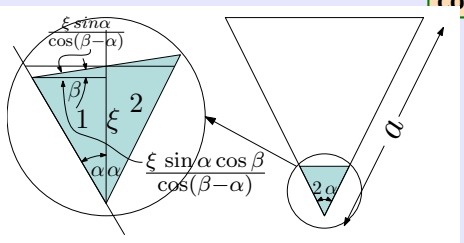


Fig. 11.6 – Triangle rotation point movement explanation.

As in Ex. 11.4 assumption is that the area (actually the volume) is constant. The volume of the triangle is

$$A = a^2 \cos\alpha \, \sin\alpha = \frac{a^2 \sin 2\alpha}{2} \tag{11.5.a}$$

where a is ,in this case, the side of the triangle. Notice that two triangles are considered. The submerged area is quarter of the total area $V_0 = 0.25\, V$ or $A_0 = 0.25\, A$. Hence, the submerged area is

$$A = \overbrace{\frac{1}{4}}^{\rho_s/\rho_\ell} \frac{a^2 \sin 2\alpha}{2} \tag{11.5.b}$$

The height of the submerged area can be evaluated as it was done before for many examples as

$$\overbrace{\frac{1}{4}}^{\rho_s/\rho_\ell} \frac{\xi^2 sin\alpha}{2} = A_1 + A_2 \tag{11.5.c}$$

where A_1 is the area of the left side triangle and A_2 is the area of the right side triangle. The dimensions of the new shape can be obtained by calculating the two zones (triangles 1 & 2). It can be noticed that triangle 1 is smaller than triangle 2 as well as their top side. The left triangle (based on the observation at Fig. 11.6) is then

$$A_1 = \frac{\xi^2}{2} \frac{\sin(\alpha)}{\cos(\theta+\alpha)} \tag{11.5.d}$$

The second area can be obtains utilizing sine law for the entire triangle to obtain the top side value. Notice, the cos is because the angle is $\pi/2 - \cdots$. With this information the other two sides can be evaluated. The area of triangle 2 is half the base times the height hence

$$A_2 = \frac{\xi^2}{2} \frac{\sin\alpha\cos\beta}{\cos(\alpha+\beta)} \tag{11.5.e}$$

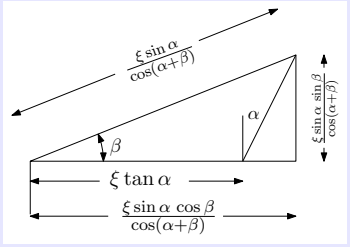


Fig. 11.7 – The upper of the right hand triangle.

End of Ex. 11.5

The total area is the sum of the two triangles and it is

$$A = \frac{\xi^2}{2}\frac{\sin(\alpha)}{\cos(\theta+\alpha)} + \frac{\xi^2}{2}\frac{\sin\alpha\cos\beta}{\cos(\alpha+\beta)} \tag{11.5.f}$$

Eq. (11.5.f) can be simplified as

$$A = \frac{\xi^2\,\sin\alpha}{2\,\cos(\alpha+\beta)}\,(1+\cos\beta) \tag{11.5.g}$$

This area has to be to the area initial area Eq. (11.5.b) thus

$$\frac{1}{4}\frac{a^2\sin 2\alpha}{\cancel{2}} = \frac{\xi^2\,\sin\alpha}{\cancel{2}\,\cos(\alpha+\beta)}\,(1+\cos\beta) \tag{11.5.h}$$

After rearmament Eq. (11.5.h) can be written as

$$\left(\frac{\xi}{a}\right)^2 = \overbrace{\frac{1}{4}}^{\rho_\ell/\rho_s}\frac{\cancel{\sin 2\alpha}^{\,2\cos\alpha}\cos(\alpha+\beta)}{\cancel{\sin\alpha}^{\,1}(1+\cos\beta)} \tag{11.5.i}$$

or after simplification Eq. (11.5.i) becomes

$$\frac{\xi}{a} = \sqrt{\frac{1}{2}\frac{\cos\alpha\,\cos(\alpha+\beta)}{1+\cos\beta}} \tag{11.5.j}$$

This found value ξ denoted as ξ^* and is used to determine the upper part of triangles (new surface) and the difference between $S_2 - S1$ is the change of the rotation location.

$$\frac{\Delta S}{a} = \frac{S_2 - S_1}{a} = \frac{\xi^*\,\sin\alpha}{\cos(\alpha+\beta)} - \frac{\xi^*\,\sin\alpha}{\cos(\beta-\alpha)} \tag{11.5.k}$$

The change is in the direction of the surface and the x component or the y component can be evaluated.

With these two angles the a graph can be drawn.

11.3.2 The change of density effect

The change of the density due to either to temperature of salinity or any other reason change the buoyancy of the floating body. Thus, this change in the location of the rotation point changes the moment of inertia relative to the rotation point. The relative change of the density can be in order a few percent points (very common) to 20% or 30%. Yet, even in the case of 3% which can in turn change the moment of inertia by 7% (hint $I \sim x^2$). This change is large enough to be noticed in the calculations. This book deals with other bodies and hence in the cases where density has a large and significant change the natural frequency also change significantly.

11.4 Transfer Mechanisms

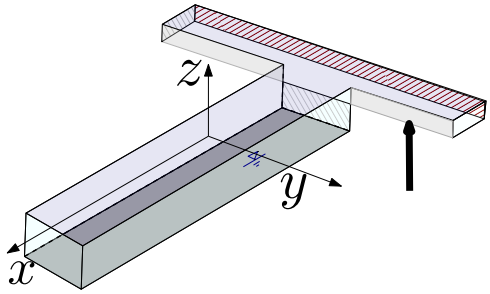


Fig. 11.8 – T shape floating to demonstrate the 3D effect The rolling creates yaw and Pitch.

The source of the original motion is generally not relevant to the transfer mechanism of the motion from one mode to another. The cause of the transfer is the geometry of the floating body[14]. One body shape that does not transfer from one mode to another is sphere. There is source of waves or other exciting forces/moments (beside the magnetic forces) that can cause transfer. Thus, the transfer of the motion from one mode to another is because symmetry (or asymmetry) of the body. Here, the transfer mechanisms are divided into two categories: one, from rotation in one dimension to rotation in another dimension, and two from rotation to linear motion (vis versa). The transfer from linear motion to linear motion is considered as the second category[15].

The movement of a floating body can be transferred due to the geometry from one mode to another. To understand this transfer consider the T–shape body depicted in Fig. 11.8. The extend part of the body is just touching the liquid. In this case, the regular marine coordinate system is adapted. If the body is rotating around the x coordinate (or around another parallel line), the buoyancy centroid changes the location when the body rotated. As before, the change in buoyancy centroid move has a component in the x direction. Yet, in this case, as opposed to the extruded body, the centroid moves to a location with in a different plane of the original z–y plane. In other words, the buoyancy centroid, $\mathcal{B}$ has two components one in y direction (as before) and new one in the x. Assuming that the body was at equilibrium, these changes in $\mathcal{B}$ centroid location creates a moment in the y axis (pitch) and also around z (yew). This T–shape effects are created by added submerged volume, however similar effects can be created by removing part of the submerged body. The meaning of removing body is that a body when rotated exhibits a depression. Generally, if the body is not symmetrical round y any rotation around the x coordinate creates rotation y coordinate and z coordinate. The gravity centroid remains the same location on the body. Thus, there is a new moment that acting the new direction. The described effect is of roll causing pitch (and yaw). Yet, the same argument can describe pitch movement causing roll. In this discussion, the source of the rotation is irrelevant.

[14] A user pointed out that movements inside the body caused by roll (for example) can also cause transfer energy.

[15] In discussion with one of the reader he point out that in the future it will be considered the third category. Time will tell.

The previous discussion dealt with the transfer from one rotation to another rotation due to the floating body asymmetry (in previous case the asymmetry was around the yz plane). It can be expended to movement of linear like heave to rotation. The main difference in this category is that the moment transferred into force. It must be mentioned that the energy during the transfer is constant and the transfer reduces the oscillations or the movement because the energy is transferred. Additionally, the transfer does not occur linearly and it depend on the change of the buoyancy centroid. The forces in the x coordinate are equal and opposite to each other and hence cancel each other. However, the forces in the y direction are are not equal and cause a moment which will results in the rolling rotation. That is, in this case the asymmetry in around y coordinate transfer energy from the heave to roll. If the floating body is not perfect symmetry it also might create movement in other directions. Note that if the body was made from straight lines going down it will not any effect on the other direction linear motion.

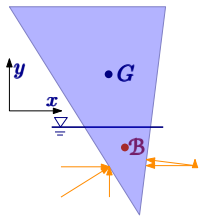


Fig. 11.9 – Extruded triangle to explain the movement transfer from Heave to Rolling.

The conceptional explanation has to get engineering expression. First the transfer from roll to pitch requires to know the change in $\mathcal{B}$ due to the roll. The calculation of the change in the x direction is done in a similar fashion that the change buoyancy centroid was ascertained for symmetrical body. These calculations are 3–D nature at least for the presentation. As this book (at least this version) is pioneer this aspect and the technique presented here to calculate is crude. Not to bogged with the mathematics the functionally assumed to be known. Consider the body shown in Fig. 11.8. First, the change in the $z-y$ of buoyancy is considered. The body is divided into many small slices of with thickness of dx. For each slice the change in buoyancy and it is denoted as dB. This value is a function of x and same time the area can be calculated it denoted as $A(x)$ (in mathematical term it referred as the weight function). The change in the y direction is

$$\Delta B(\theta)_x = \frac{1}{V_0}\int_{V_0} \Delta B(x)A(x)dx \tag{11.10}$$

The change in the y direction is

$$\Delta B(\theta)_y = \frac{1}{V_0}\int_{V_0} y\, A(x)_{at\ new\ \theta} dx \tag{11.11}$$

It can be notice that change in the x is not relevant to the y direction.

Example 11.6: Heave motion **Level: Intermediate**

Calculate the change in the y direction of the body provided in the figure.

11.5 Transfer Linear Motion to Rotating Motion

11.6 Appendix Double Pendulum: Simplifications

In this section a discussion on the double pendulum which ship similar too. Essentially ship is not a regular pendulum but it is a what is known as double pendulum with point mass. These point do not represent the ship nor the circular motion of the point 1. A gradual progression will be followed since material is written for a textbook. That is, first introduction to a circular double pendulum with point masses later a discussion about the circular double pendulum with mass distribution (moment of inertia instead of mass) will introduced.

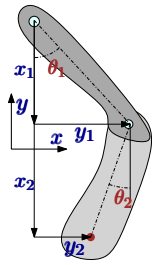


Fig. 11.10 – Simple double circular pendulum with point mass.

Later the next stage is to deal with the circular movement to linear where x_1 is related to θ_1.

11.6.1 Double Pendulum with Point Masses

The location of point 1 is

$$\begin{aligned} x_1 &= L_1 \sin\theta_1 \\ y_1 &= -L_1 \cos\theta_1 \end{aligned} \tag{11.12}$$

The location of point 2 is

$$\begin{aligned} x_2 &= x_1 + L_2 \sin\theta_2 \\ y_2 &= y_1 - L_2 \cos\theta_2 \end{aligned} \tag{11.13}$$

Taking the derivative to obtain the velocity

$$\begin{aligned} x_1' &= \theta_1' L_1 \cos\theta_1 \\ y_1' &= \theta_1' L_1 \sin\theta_1 \end{aligned} \tag{11.14}$$

In the same way for the second point

$$\begin{aligned} x_2' &= x1' + \theta_2' L_2 \cos\theta_2 \\ y_2' &= y_1' + \theta_2' L_2 \sin\theta_2 \end{aligned} \tag{11.15}$$

The second derivative is

$$\begin{aligned} x_1'' &= {\theta_1'}^2 L_1 \sin\theta_1 + \theta_1'' L_1 \cos\theta_1 \\ y_1'' &= {\theta_1'}^2 L_1 1 \cos\theta_1 + \theta_1'' L_1 \sin\theta_1 \end{aligned} \tag{11.16}$$

the second derivative is for the second point

$$\begin{aligned} x_2'' &= x_1'' {\theta_2'}^2 \, L_2 \, \sin\theta_2 + \theta_2'' L_2 \, \cos\theta_2 \\ y_2'' &= y_1'' + {\theta_2'}^2 \, L_2 \, \cos\theta_2 + \theta_2'' L_2 \, \sin\theta_2 \end{aligned} \tag{11.17}$$

where T denotes the tension and m denotes the mass and standard stationary equation can be written. The forces that act on mass 1 and mass 2 are the tension in the rods and the gravity.

$$\begin{aligned} m_1 \, x_1'' &= -T_1 \, \sin\theta_1 + T_2 \sin\theta_2 \\ m_1 \, y_1'' &= T_1 \, \cos\theta_1 T_2 \cos\theta_2 - m_1 \, g \end{aligned} \tag{11.18}$$

and for the second point

$$m_2 \, x_2'' = T_2 \sin\theta_2 \tag{11.19}$$

$$m_2 \, y_2'' = T_2 \cos\theta_2 - m_2 \, g \tag{11.20}$$

Basically there are 4 equations and 4 unknowns $(T_1, T_2, \theta_1, \text{ and } \theta_2)$. Notice that $x_1, x_2, y_1, \text{ and } y_2$ are a simple function of L and θ with the right indexes. After the elimination or conversion of "dummy" variables (T_1, T_2) results in two equations with unknowns which can be solved by numerical methods like Runge Kutta method. No recommendations on the specific methods is provided.

The results are strongly depend on the initial conditions. There several conclusions can be drown from the results which are very relevant to floating body stability. The chief conclusion is that standard natural frequency does not exist. The results are more chaotic that one expect. For small angle the behavior is close to a simple pendulum. Even with going through more complicate examination one can see that situation get more complex (this with including the change of moment of inertia and other factors).

11.6.2 Double Pendulum with Circular Motion with Mass Distribution

This section is out reach for many of the readers because it involve advance physics and concept like Lagrangian. These concepts are used because the complication of writing the governing equations. The usage of Lagrangian provides the governing equations in a standard form. Equations presented in a standard form and thus element the need to usage of vectors. With the Lagrangian methods there is no need to solve for accelerations or a need to solve for inertial velocities. So simple demonstration of usage of Lagrangian for simple pendulum is presented. The Lagrangian function is defined as the difference between the total kinetic energy, T and total potential energy U of system.

$$L = T - U \tag{11.21}$$

For simple pendulum with point mass with arm length, ℓ the potential energy is relative the lowest point and for small angle. The velocity is

$$v = \ell \, \theta \tag{11.22}$$

The total kinetic energy is

$$T = \frac{m\,v^2}{2} = \frac{m\,\ell^2\dot{\theta}^2}{2} \tag{11.23}$$

The potential energy, U, in case of simple pendulum, depends only on the height. For simplicity the potential energy is assigned $U(\theta = 0) = 0$ Thus the potential energy can be written as

$$U = y\,m\,g = \ell\,(1 - \cos\theta)\,m\,g \tag{11.24}$$

With these two terms the Lagrangian is explicitly can written as

$$L = T - U = \frac{g\,m\,(\ell\,\dot{\theta})^2}{2} - \ell\,(1 - \cos\theta)\,m\,g \tag{11.25}$$

For the next part The governing equation is obtained utilizing for zero non–conservative forces by

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} = 0 \tag{11.26}$$

substituting L into Eq. (11.26) yields

$$m\,\ell^2\,\theta'' + m\,g\,\ell\,\sin\theta = 0 \longrightarrow \theta'' + \frac{g}{\ell}\,\sin\theta = 0 \tag{11.27}$$

which is the standard equation for simple pendulum.

It can be noticed that coordinate system change some what. That the distances are measured to the center of the leg which has mass and moment of inertia. It is assumed that the moment of inertia is given. The distance x_1 is this only to half of the leg and some can be said about the y_1. The distances can be express as

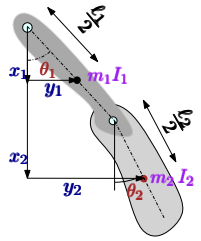


Fig. 11.11 – Double pendulum linear motion in circular motion with disperse mass.

$$x_1 = \frac{\ell}{2}\sin\theta_1 \tag{11.28}$$

$$y_1 = -\frac{\ell}{2}\cos\theta_1 \tag{11.29}$$

It is assumed that rod are evenly distribute otherwise new dimension can be introduced.

$$\begin{aligned} x_2 &= \ell_1\,\sin\theta_1 + \ell_2\frac{\sin\theta_2}{2} \\ y_2 &= -\ell_1\cos\theta_1 - \ell_2\frac{\cos\theta_2}{2} \end{aligned} \tag{11.30}$$

The kinetic energy can be expressed as

$$T = \frac{m_1 {v_1}^2}{2} + \frac{m_2 {v_2}^2}{2} + \frac{I_1 {\theta'_1}^2}{2} + \frac{I_2 {\theta'_2}^2}{2} \tag{11.31}$$

The potential energy is

$$U = g\, y_1\, m_1 + g\, y_2\, m_2 \tag{11.32}$$

Thus the Lagrangian is

$$L = T - U = \frac{m_1 {v_1}^2}{2} + \frac{m_2 {v_2}^2}{2} + \frac{I_1 {\theta'_1}^2}{2} + \frac{I_2 {\theta'_2}^2}{2} - m_1 g y_1 - m_2 g y_2. \tag{11.33}$$

Assuming no external forces the governing equations are obtained from

$$\begin{aligned} \frac{d}{dt}\frac{\partial L}{\partial {\theta'}_1} - \frac{\partial L}{\partial \theta_2} &= 0 \\ \frac{d}{dt}\frac{\partial L}{\partial {\theta'}_2} - \frac{\partial L}{\partial \theta_1} &= 0 \end{aligned} \tag{11.34}$$

At this stage the continuation can be found the literature as it establish issue. The main point was to establish the method and demonstrate the ship rotation similar to double pendulum but with a twist in which the relationship between the hinge to physics is determined from ship determined.

11.6.3 Double Pendulum with Linear Motion

The relation between y_1 (see Fig. 11.12) and θ_1 is determined from the cross section (for 2D simpler and it more complex for 3D.). This relationship is independent of the angular velocity and simply related to angle itself θ_1 and related based on geometrical parameters. This relationship obtained by following the instructions in chapter. Given this relationship only one body as control mass is needed.

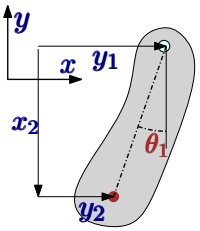


Fig. 11.12 – Double pendulum linear motion with disperse mass.

11.7 Appendix: The Added Properties

The explanation of the added properties is very complicated at least the mathematical part of it [16] This material way out scope of this book yet, it is here just to provide a

[16] This author was a teaching assistance long ago for Touvia Miloh (Landweber and Miloh 1980) who was teaching class on the added properties. This knowledge is from time of being a teaching assistance for that class. The never researched in this area.

justification why this term is added. This addition due to the realization that a body moves in a liquid it drags additional (therefore the word added) material/mass (in the liquid phase). This addition strongly depends on the geometry of the body and the its acceleration. For example, a sphere moving steady state has coefficient of add mass is 0.5 which can be larger than the momentum of the body itself. This value is, roughly speaking, calculated by the singular points of potential flow. The added moment of inertia of the ship body depends on the angle of the ship and the acceleration.

11.8 Pendulum With Moving Hinge Point

11.9 Change of Center of Rotation

At first the discussion deals with 2D cases as they simpler as compared to 3D. The center of ration is at the middle of the liquid line (for two dimensional body). However, the change of the center of rotation is in the average of addition to right and the subtraction to the left assume counter–clock turn. In this derivation it is assumed that the body is extruded that is it only 2–D and the angle is very small. Later it will derive for measurable angles. In this derivations the body is not assumed to symmetrical. The total change is made of the change to the right and to the left. The change to the right is

$$\Delta x_r = \frac{dA_r}{dh} \tag{11.35}$$

The Δx_r represents the change to the right. If the body is symmetrical that total change is $\Delta x = \Delta x_r = \Delta x_l$. The change to the left is Δx_l is

$$\Delta x_l = -\frac{dA_l}{dh} \tag{11.36}$$

The Δx_l represents the change of the contact point of surface to the left. The total change is then

$$\Delta x_{total} = \frac{dA_r}{dh} - \frac{dA_l}{dh} \tag{11.37}$$

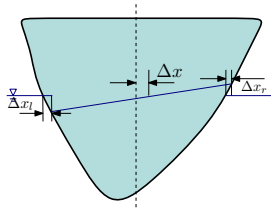


Fig. 11.13 – The derivative of the change of the rotation points.

12

Dimensional Analysis

This chapter is dedicated to my adviser, Dr. E.R.G. Eckert.
Genick Bar-Meir

12.1 Introductory Remarks

The reasons that this chapter is included in this book to eliminate the tables for various situation but to have dimensionless presentation so tables should be included in the equations them self. For example, in Metacenter method the parameter GM is not given in dimensionless form. What is the meaning of GM 1[m] while it say the ship is stable and does not indicate if it is very stable or not. One has to get additional information. The indication that this question of stability is has relatively small of parameters which indicate that it has to none–dimensionalized.

What is dimensionless analysis? Dimensional analysis refers to techniques dealing with units or conversion to a unitless system. The definition of dimensional analysis is not consistent in the literature which span over various fields (mostly in the thermo–fluid field) (Buckingham 1914; Buckingham 1915b; Buckingham 1915a). Possible topics that dimensional analysis deals with are consistency of the units, change order of magnitude, applying from the old and known to unknown (see the Book of Ecclesiastes), and creation of group parameters without any dimensions. In this chapter, the focus is on the creation the creation of dimensionless groups. These techniques gave birth to dimensional parameters which have a great scientific importance (Görtler 1975; Lang-

haar 1980). Since the 1940s[1], the dimensional analysis is taught and written in all fluid, heat and mass transfer textbooks. The approach or the technique used in some of these books is referred to as Buckingham–π–theory. The π–theory was coined by Buckingham. However, there is another technique which is referred to in the literature as the Nusselt's method. Both these methods attempt to reduce the number of parameters which affect the problem and reduce the labor in solving the problem. The key in these techniques lays in the fact of consistency of the dimensions of any possible governing equation(s) and the fact that some dimensions are reoccurring. The Buckingham–π goes further and no equations are solved and even no knowledge about these equations is required. In Buckingham's technique only the dimensions or the properties of the problem at hand are analyzed. This author is aware of only a single class of cases were Buckingham's methods is useful and or can solve the problem namely the pendulum class problem (and similar).

The dimensional analysis was independently developed by Nusselt and improved by his students/co–workers (Schmidt, Eckert) in which the governing equations are used as well. Thus, more information is put into the problem and thus a better understanding on the dimensionless parameters is extracted. The advantage or disadvantage of these similar methods depend on the point of view. The Buckingham–π technique is simpler while Nusselt's technique produces a better result. Sometime, the simplicity of Buckingham's technique yields insufficient knowledge or simply becomes useless. When no governing equations are found, Buckingham's method has usefulness. It can be argued that these situations really do not exist in the Thermo–Fluid field. Nusselt's technique is more cumbersome but more precise and provide more useful information. Both techniques are discussed in this Chapter. The advantage of the Nusselt's technique are: a) compact presentation, b)knowledge what parameters affect the problem, c) easier to extent the solution to more general situations. In very complex problems both techniques suffer from in inability to provide a significant information on the effective parameters such multi–phase flow etc.

It has to be recognized that the dimensional analysis provides answer to what group of parameters affecting the problem and not the answer to the problem. In fact, there are fields in thermo–fluid where dimensional analysis, is recognized as useless. For example, the area of multiphase flows there is no solution based on dimensionless parameters (with the exception of the rough solution of Martinelli). In the Buckingham's approach it merely suggests the number of dimensional parameters based on a guess of all parameters affecting the problem. Nusselt's technique provides the form of these dimensionless

[1]The history of dimensional analysis is complex. Several scientists used this concept before Buckingham and Nusselt (see below history section). Their work culminated at the point of publishing the paper Buckingham's paper and independently constructed by Nusselt. It is interesting to point out that there are several dimensionless numbers that bear Nusselt and his students name, Nusselt number, Schmidt number, Eckert number. There is no known dimensionless number which bears Buckingham name. Buckingham's technique is discussed and studied in Fluid Mechanics while almost completely ignored by Heat and Mass Transfer researchers and their classes. Furthermore, in many advance fluid mechanics classes Nusselt's technique is used and Buckingham's technique is abandoned. Perhaps this fact can be attributed to tremendous influence Nusselt and his students had on the heat transfer field. Even, this author can be accused for being bias as the Eckert's last student. However, this author observed that Nusselt's technique is much more effective as it will demonstrated later.

parameters, and the relative relationship of these parameters.

12.1.1 Brief History

The idea of experimentation with a different, rather than the actual, dimension was suggested by several individuals independently. Some attribute it to Newton (1686) who coined the phrase of "great Principle of Similitude." Later, Maxwell a Scottish Physicist played a major role in establishing the basic units of mass, length, and time as building blocks of all other units. Another example, John Smeaton (8 June 1724–28 October 1792) was an English civil and mechanical engineer who study relation between propeller/wind mill and similar devices to the pressure and velocity of the driving forces. Jean B. J. Fourier (1768-1830) first attempted to formulate the dimensional analysis theory. This idea was extend by William Froude (1810-1871) by relating the modeling of open channel flow and actual body but more importantly the relationship between drag of models to actual ships. While the majority of the contributions were done by thermo–fluid guys the concept of the equivalent or similar propagated to other fields. Aiméem Vaschy, a German Mathematical Physicist (1857–1899), suggested using similarity in electrical engineering and suggested the Norton circuit equivalence theorems. Rayleigh probably was the first one who used dimensional analysis (1872) to obtain the relationships between the physical quantities (see the question why the sky is blue story).
Osborne Reynolds (1842–1912) was the first to derive and use dimensionless parameters to analyze experimental data. Riabouchunsky[2] proposed of relating temperature by molecules velocity and thus creating dimensionless group with the byproduct of compact solution (solution presented in a compact and simple form).
Buckingham culminated the dimensional analysis and similitude and presented it in a more systematic form. In the about the same time (1915, Wilhelm Nusselt (November 25, 1882 – September 1, 1957), a German engineer, developed the dimensional analysis (proposed the principal parameters) of heat transfer without knowledge about previous work of Buckingham.

12.1.2 Theory Behind Dimensional Analysis

In chemistry it was recognized that there are fundamental elements that all the material is made from (the atoms). That is, all the molecules are made from a combination of different atoms. Similarly to this concept, it was recognized that in many physical systems there are basic fundamental units which can describe all the other dimensions or units in the system. For example, isothermal single component systems (which does not undergo phase change, temperature change and observed no magnetic or electrical effect) can be described by just basic four physical units. The units or dimensions are, time, length, mass, quantity of substance (mole). For example, the dimension or the units of force can be constructed utilizing Newton's second law i.e. mass times acceleration $\longrightarrow m\,a = M\,L/t^2$. Increase of degree of freedom, allowing this system

[2]Riabouchunsky, Nature Vol 99 p. 591, 1915

to be non–isothermal will increase only by one additional dimension of temperature, θ. These five fundamental units are commonly the building blocks for most of the discussion in fluid mechanics (see Table of basic units 13.3).

Table 12.1 – Basic Units of Two Common Systems

Standard System			Old System		
Name	Letter	Units	**Name**	Letter	Units
Mass	M	$[kg]$	**Force**	F	$[N]$
Length	L	$[m]$	**Length**	L	$[m]$
Time	t	$[sec]$	**Time**	t	$[sec]$
Temperature	θ	$[^\circ\mathrm{C}]$	**Temperature**	T	$[^\circ\mathrm{C}]$
Additional Basic Units for Magnetohydrodynamics					
Electric Current	A	$[A]mpere$	**Electric Current**	A	$[A]mpere$
Luminous Intensity	cd	$[cd]$ candle	**Luminous Intensity**	cd	$[cd]$ candle
Chemical Reactions					
Quantity of substance	$\mathfrak{M}$	mol	**Quantity of substance**	$\mathfrak{M}$	mol

The choice of these basic units is not unique and several books and researchers suggest a different choice of fundamental units. One common selection is substituting the mass with the force in the previous selection (F, t, L, mol, Temperature). This author is not aware of any discussion on the benefits of one method over the other method. Yet, there are situations in which first method is better than the second one while in other situations, it can be the reverse. Other selections are possible but not common and, at the moment, will not be discussed here.

Example 12.1: What are the units of Level: Easy

What are the units of force when the basic units are: mass, length, time, temperature (M, L, t, θ)? What are the units of mass when the basic units are: force, length, time, temperature (F, L, t, T)? Notice the different notation for the temperature in the two systems of basic units. This notation has no significance but for historical reasons remained in use.

Solution

End of Ex. 12.1

These two systems are related as the questions are the reversed of each other. The connection between the mass and force can be obtained from the simplified Newton's second law $F = m\,a$ where F is the force, m is the mass, and a is the acceleration. Thus, the units of force are

$$F = \frac{M\,L}{t^2} \tag{12..a}$$

For the second method the unit of mass are obtain from Equation (12..a) as

$$M = \frac{F\,t^2}{L} \tag{12..b}$$

The number of fundamental or basic dimensions determines the number of the combinations which affect the physical[3] situations. The dimensions or units which affect the problem at hand can be reduced because these dimensions are repeating or reoccurring. The Buckingham method is based on the fact that all equations must be consistent with their units. That is the left hand side and the right hand side have to have the same units. Because they have the same units the equations can be divided to create unitless equations. This idea alludes to the fact that these unitless parameters can be found without any knowledge of the governing equations. Thus, the arrangement of the effecting parameters in unitless groups yields the affecting parameters. These unitless parameters are the dimensional parameters. The following trivial example demonstrates the consistency of units

Example 12.2: Newton's law dimensions Level: Easy

Newton's equation has two terms that related to force $F = m\,a + \dot{m}\,U$. Where F is force, m is the mass, a is the acceleration and dot above $\dot{m}$ indicating the mass derivative with respect to time. In particular case, this equation get a form of

$$F = m\,a + 7 \tag{12..c}$$

where 7 represent the second term. What are the requirement on equation (12..c)?

Solution

Clearly, the units of $[F]$, $m\,a$ and 7 have to be same. The units of force are $[N]$ which is defined by first term of the right hand side. The same units force has to be applied to 7 thus it must

[3] The dimensional analysis also applied in economics and other areas and the statement should reflect this fact. However, this book is focused on engineering topics and other fields are not discussed.

be in $[N]$.

End of Ex. 12.2

Suppose that there is a relationship between a quantity a under the question and several others parameters which either determined from experiments or theoretical consideration which is of the form

$$D = f(a_1, a_2, \cdots, a_i, \cdots, a_n) \tag{12.1}$$

where D is dependent parameters and $a_1, a_2, \cdots, a_i, \cdots, a_n$ are have independent dimensions. From these independent parameters $a_1, a_2, \cdots, a_i$ have independent dimensions (have basic dimensions). This mean that all the dimensions of the parameters $a_{i+1}, \cdots, a_n$ can be written as combination of the the independent parameters $a_1, a_2, \cdots, a_i$. In that case it is possible to write that every parameter in the later set can written as dimensionless

$$\frac{a_{i+1}}{{a_1}^{p_1}, {a_2}^{p_2}, \cdots, {a_i}^{p_i}} = \text{dimensionless} \tag{12.2}$$

The "non–basic" parameter would be dimensionless when divided by appropriately and selectively chosen set of constants $p_1, p_2, \cdots, p_i$.

Example 12.3: Clamping force

Level: Easy

In a experiment, the clamping force is measured. It was found that the clamping force depends on the length of experimental setup, velocity of the upper part, mass of the part, height of the experimental setup, and leverage the force is applied. Chose the basic units and dependent parameters. Show that one of the dependent parameters can be normalized.

Solution

The example suggest that the following relationship can be written.

$$F = f(L, U, H, \tau, m) \tag{12..d}$$

The basic units in this case are in this case or length, mass, and time. No other basic unit is need to represent the problem. Either L, H, or τ can represent the length. The mass will be represented by mass while the velocity has to be represented by the velocity (or some combination of the velocity). Hence a one possible choice for the basic dimension is L, m, and U. Any of the other Lengths can be represented by simple division by the L. For example

$$\text{Normalize parameter} = \frac{H}{L} \tag{12..e}$$

End of Ex. 12.3

Or the force also can be normalized as

$$\text{Another Normalize parameter} = \frac{F}{m\,U^2\,L^{-1}} \qquad (12..f)$$

The acceleration can be any part of acceleration component such as centrifugal acceleration. Hence, the force is mass times the acceleration.

The relationship (12.1) can be written in the light of the above explanation as

$$\frac{D}{a_1{}^{p_1}, a_2{}^{p_2}, \cdots, a_i{}^{p_1}} = F\left(\frac{a_{i+1}}{a_1{}^{p_{i+1,1}},\, a_2{}^{p_{i+1,2}}, \cdots,\, a_i{}^{p_{i+1,i}}}, \cdots, \frac{a_n}{a_n{}^{p_{n,1}},\, a_n{}^{p_{n,2}}, \cdots,\, a_n{}^{p_{n,i}}}\right) \qquad (12.3)$$

where the indexes of the power p on the right hand side are single digit and the double digits on the on the right hand side. While this "proof" shows the basic of the Buckingham's method it actually provides merely the minimum number of the dimension parameters. In fact, this method entrenched into the field while in most cases provides incomplete results. The fundamental reason for the erroneous results is because the fundamental assumption of equation (12.1). This method provides a crude tool of understanding.

12.1.3 Dimensional Parameters Application for Experimental Study

The solutions for any situations which are controlled by the same governing equations with same boundary conditions regardless of the origin the equation. The solutions are similar or identical regardless to the origin of the field no matter if the field is physical, or economical, or biological. The Buckingham's technique implicitly suggested that since the governing equations (in fluid mechanics) are essentially are the same, just knowing the parameters is enough the identify the problem. This idea alludes to connections between similar parameters to similar solution. The non–dimensionalization i.e. operation of reducing the number affecting parameters, has a useful by–product, the analogy in other words, the solution by experiments or other cases. The analogy or similitude refers to understanding one phenomenon from the study of another phenomenon. This technique is employed in many fluid mechanics situations. For example, study of compressible flow (a flow where the density change plays a significant part) can be achieved by study of surface of open channel flow. The compressible flow is also similar to traffic on the highway. Thus for similar governing equations if the solution exists for one case it is a solution to both cases.

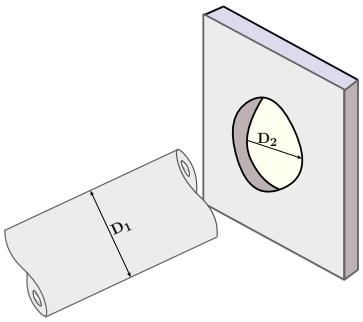


Fig. 12.1 – Fitting rod into a hole.

The analogy can be used to conduct experiment in a cheaper way and/or a safer way. Experiments in different scale than actual dimensions can be conducted for cases where the actual dimensions are difficult to handle. For example, study of large air planes can done on small models. On the other situations, larger models are used to study small or fast situations. This author believes that at the present the Buckingham method has extremely limited use for the real world and yet this method is presented in the classes on fluid mechanics. Thus, many examples on the use of this method will be presented in this book. On the other hand, Nusselt's method has a larger practical use in the real world and therefore will be presented for those who need dimensional analysis for the real world. Dimensional analysis is useful also for those who are dealing with the numerical research/calculation. This method supplement knowledge when some parameters should be taken into account and why.

Fitting a rod into a circular hole (see Figure 12.1) is an example how dimensional analysis can be used. To solve this problem, it is required to know two parameters; 1) the rode diameter and 2) the diameter of the hole. Actually, it is required to have only one parameter, the ratio of the rode diameter to the hole diameter. The ratio is a dimensionless number and with this number one can tell that for a ratio larger than one, the rode will not enter the hole; and a ratio smaller than one, the rod is too small. Only when the ratio is equal to one, the rode is said to be fit. This presentation allows one to draw or present the situation by using only one coordinate, the radius ratio. Furthermore, if one wants to deal with tolerances, the dimensional analysis can easily be extended to say that when the ratio is equal from 0.99 to 1.0 the rode is fitting, and etc. If one were to use the two diameters description, further significant information will be needed. In the preceding simplistic example, the advantages are minimal. In many real problems this approach can remove clattered views and put the problem into focus. Throughout this book the reader will notice that the systems/equations in many cases are converted to a dimensionless form to augment understanding.

12.1.4 The Pendulum Class Problem

The only known problem that dimensional analysis can solve (to some degree) is the pendulum class problem. In this section several examples of the pendulum type problem are presented. The first example is the classic Pendulum problem.

Example 12.4: Dimensionless Pendulum **Level: Easy**

Derive the relationship for the gravity [g], frequency [ω] and length of pendulum [ℓ]. Assume that no other parameter including the mass affects the problem. That is, the relationship can be expressed as

$$\omega = f(\ell, g) \qquad (12..g)$$

Notice in this problem, the real knowledge is provided, however in the real world, this knowledge is not necessarily given or known. Here it is provided because the real solution is already known from standard physics classes.[4]

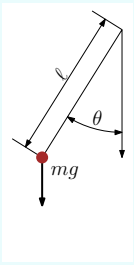


Fig. 12.2 – Figure for example (12.4).

Solution

The solution technique is based on the assumption that the indexical form is the appropriate form to solve the problem. The Indexical form

$$\omega = C_1 \times \ell^a g^b \qquad (12..h)$$

The solution functional complexity is limited to the basic combination which has to be in some form of multiplication of ℓ and g in some power. In other words, the multiplication of $\ell\, g$ have to be in the same units of the frequency units. Furthermore, assuming, for example, that a trigonometric function relates ℓ and g and frequency. For example, if a sin function is used, then the functionality looks like $\omega = \sin(\ell\, g)$. From the units point of view, the result of operation not match i.e. $(sec \neq \sin(sec))$. For that reason the form in equation (12..h) is selected. To satisfy equation (12..h) the units of every term are examined and summarized the following table.

Table 12.2 – Units of the Pendulum Parameters

Parameter	Units	Parameter	Units	Parameter	Units
ω	t^{-1}	ℓ	L^1	g	$L^1 t^{-2}$

End of Ex. 12.4

Thus substituting of the Table 12.2 in equation (12..h) results in

$$t^{-1} = C_1 \left(L^1\right)^a \left(L^1\, t^{-2}\right)^b \Longrightarrow L^{a+b} t^{-2\,b} \tag{12..i}$$

after further rearrangement by multiply the left hand side by L^0 results in

$$L^0 t^{-1} = C\, L^{a+b} t^{-2\,b} \tag{12..j}$$

In order to satisfy equation (12..j), the following must exist

$$0 = a + b \quad and \quad -1 = \tfrac{-2}{b} \tag{12..k}$$

The solution of the equations (12..k) is $a = -1/2$ and $b = -1/2$.

What was found in this example is the form of the solution's equation and frequency. Yet, the functionality e.g. the value of the constant was not found. The constant can be obtained from experiment for plotting ω as the abscissa and $\sqrt{\ell/g}$ as ordinate.

According to some books and researchers, this part is the importance of the dimensional analysis. It can be noticed that the initial guess merely and actually determine the results. If, however, the mass is added to considerations, a different result will be obtained. If the guess is relevant and correct then the functional relationship can be obtained by experiments.

12.2 Buckingham–π–Theorem

All the physical phenomena that is under the investigation have n physical effecting parameters such that

$$F_1(q_1, q_2, q_3, \cdots, q_n) = 0 \tag{12.4}$$

where q_i is the "i" parameter effecting the problem. For example, study of the pressure difference created due to a flow in a pipe is a function of several parameters such

$$\Delta P = f(L,\, D,\, \mu,\, \rho,\, U) \tag{12.5}$$

In this example, the chosen parameters are not necessarily the most important parameters. For example, the viscosity, μ can be replaced by dynamic viscosity, ν. The choice is made normally as the result of experience and it can be observed that ν is a function of μ and ρ. Finding the important parameters is based on "good fortune" or perhaps intuition. In that case, a new function can be defined as

$$F(\Delta P, L, D, \mu, \rho, U) = 0 \tag{12.6}$$

[4] The reader can check if the mass is assumed to affect the problem then, the result is different.

Again as stated before, the study of every individual parameter will create incredible amount of data. However, Buckingham's[5] methods suggested to reduce the number of parameters. If independent parameters of same physical situation is m thus in general it can be written as

$$F_2(\pi_1, \pi_2, \pi_3, \cdots, \pi_m) = 0 \tag{12.7}$$

If there are n variables in a problem and these variables contain m primary dimensions (for example M, L, T), then the equation relating all the variables will have (n-m) dimensionless groups.

There are 2 conditions on the dimensionless parameters:

1. Each of the fundamental dimensions must appear in at least one of the m variables
2. It must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

In the case of the pressure difference in the pipe (Equation (12.6)) there are 6 variables or $n = 6$. The number of the fundamental dimensions is 3 that is $m = 3$ ([M], [L], [t]) The choice of fundamental or basic units is arbitrary in that any construction of these units is possible. For example, another combination of the basic units is time, force, mass is a proper choice. According to Buckingham's theorem the number of dimensionless groups is $n - m = 6 - 3 = 3$. It can be written that one dimensionless parameters is a function of two other parameter such as

$$\pi_1 = f(\pi_2, \pi_3) \tag{12.8}$$

If indeed such a relationship exists, then, the number of parameters that control the problem is reduced and the number of experiments that need to be carried is considerably smaller. Note, the π–theorem does not specify how the parameters should be selected nor what combination is preferred.

12.2.1 Construction of the Dimensionless Parameters

In the construction of these parameters it must be realized that every dimensionless parameters has to be independent. The meaning of independent is that one dimensionless parameter is not a multiply or a division of another dimensional parameter. In the above example there are three dimensionless parameters which required of at least one of the physical parameter per each dimensionless parameter. Additionally, to make these dimensionless parameters independent they cannot be multiply or division of each other.

For the pipe problem above, ℓ and D have the same dimension and therefore both cannot be chosen as they have the same dimension. One possible combination

[5] E. Buckingham, "Model Experiments and the Forms of Empirical Equations," Transactions of the American Society of Mechanical Engineers, Vol. 37, 1915.

is of D, U and ρ are chosen as the recurring set. The dimensions of these physical variables are: $D = [L^1]$, velocity of $U = [L\,t^{-1}]$ and density as $\rho = [M\,L^{-3}]$. Thus, the first term D can provide the length, $[L]$, the second term, U, can provide the time $[t]$, and the third term, ρ can provide the mass $[M]$. The fundamental units, L, t, and M are length, time and mass respectively. The fundamental units can be written in terms of the physical units. The first term L is the described by D with the units of $[L]$. The time, $[t]$, can be expressed by D/U. The mass, $[M]$, can be expressed by $\rho\,D^3$. Now the dimensionless groups can be constructed by looking at the remaining physical parameters, ΔP, D and μ. The pressure difference, ΔP, has dimensions of $[M\,L^{-1}\,t^{-2}]$ Therefore, $\Delta P\,M^{-1}\,L\,t^2$ is a dimensionless quantity and these values were calculated just above this line. Thus, the first dimensionless group is

$$\pi_1 = \overbrace{\Delta P}^{[M\,L^{-1}\,t^{-2}]}\ \overbrace{\frac{1}{\rho\,D^3}}^{[M^{-1}]}\ \overbrace{D}^{[L]}\ \overbrace{\frac{D^2}{U^2}}^{[t^2]} = \overbrace{\frac{\Delta P}{\rho\,U^2}}^{unitless} \tag{12.9}$$

The second dimensionless group (using D) is

$$\pi_2 = \overbrace{D}^{[L]}\ \overbrace{\ell^{-1}}^{[L^{-1}]} = \frac{D}{L} \tag{12.10}$$

The third dimensionless group (using μ dimension of $[M\,L^1\,t^{-1}]$) and therefore dimensionless is

$$\pi_3 = \mu\ \overbrace{\frac{1}{D^3\,\rho}}^{[M^{-1}]}\ \overbrace{D}^{[L]}\ \overbrace{\frac{D}{U}}^{[t]} = \frac{\mu}{D\,U\,\rho} \tag{12.11}$$

This analysis is not unique and there can be several other possibilities for selecting dimensionless parameters which are "legitimately" correct for this approach.

There are roughly three categories of methods for obtaining the dimensionless parameters. The first one solving it in one shot. This method is simple and useful for a small number of parameters. Yet this method becomes complicated for large number of parameters. The second method, some referred to as the building blocks method, is described above. The third method is by using dimensional matrix which is used mostly by mathematicians and is less useful for engineering purposes.

The second and third methods require to identification of the building blocks. These building blocks are used to construct the dimensionless parameters. There are several requirements on these building blocks which were discussed on page 165. The main point that the building block unit has to contain at least the basic or fundamental unit. This requirement is logical since it is a building block. The last method is mostly used by mathematicians which leads and connects to linear algebra. The fact that this method used is the hall mark that the material was written by mathematician. Here, this material will be introduced for completeness sake with examples and several terms associated with this technique.

12.2.2 Similarity and Similitude

One of dimensional analysis is the key point is the concept that the solution can be obtained by conducting experiments on similar but not identical systems. The analysis here suggests and demonstrates[6] that the solution is based on several dimensionless numbers. Hence, constructing experiments of the situation where the same dimensionless parameters obtains could, in theory, yield a solution to problem at hand. Thus, knowing what are dimensionless parameters should provide the knowledge of constructing the experiments.

In this section deals with these similarities which in the literature some refer as analogy or similitude. It is hard to obtain complete similarity. Hence, there is discussion how similar the model is to the prototype. It is common to differentiate between three kinds of similarities: geometric, kinetics, and dynamic. This characterization started because historical reasons and it, some times, has merit especially when applying Buckingham's method. In Nusselt's method this differentiation is less important.

Geometric Similarity

One of the logical part of dimensional analysis is how the experiences should be similar to actual body they are supposed to represent. This logical conclusion is an add–on and this author is not aware of any proof to this requirement based on Buckingham's methods. Ironically, this conclusion is based on Nusselt's method which calls for the same dimensionless boundary conditions. Again, Nusselt's method, sometimes or even often, requires similarity because the requirements to the boundary conditions. Here[7] this postulated idea is adapted.

Under this idea the prototype area has to be square of the actual model or

$$\frac{A_p}{A_m} = \left(\frac{\ell_{1prototype}}{\ell_{1model}}\right)^2 = \left(\frac{\ell_{2p}}{\ell_{2m}}\right)^2 \tag{12.12}$$

where ℓ_1 and ℓ_2 are the typical dimensions in two different directions and subscript p refers to the prototype and m to the model. Under the same argument the volumes change with the cubes of lengths.

In some situations, the model faces inability to match two or more dimensionless parameters. In that case, the solution is to sacrifice the geometric similarity to minimize the undesirable effects. For example, river modeling requires to distort vertical scales to eliminate the influence of surface tension or bed roughness or sedimentation.

Kinematic Similarity

The perfect kinetics similarity is obtained when there are geometrical similarity and the motions of the fluid above the objects are the same. If this similarity is not possible, then the desire to achieve a motion "picture" which is characterized by ratios

[6] This statement is too strong. It has to be recognized that the results are as good as the guessing which in most cases is poor.

[7] Because this book intend to help students to pass their exams, this book present what most instructors required. It well established that this over–strict requirement and under Nusselt's method it can be overcome.

of corresponding velocities and accelerations is the same throughout the actual flow field. It is common in the literature, to discuss the situations where the model and prototype are similar but the velocities are different by a different scaling factor.

The geometrical similarity aside the shapes and counters of the object it also can requires surface roughness and erosion of surfaces of mobile surfaces or sedimentation of particles surface tensions. These impose demands require a minimum on the friction velocity. In some cases the minimum velocity can be $U_{min} = \sqrt{\tau_w/\rho}$. For example, there is no way achieve low Reynolds number with thin film flow.

Dynamics Similarity

The dynamic similarity has many confusing and conflicting definitions in the literature. Here this term refers to similarity of the forces. It follows, based on Newton's second law, that this requires similarity in the accelerations and masses between the model and prototype.

Example 12.5: Nusselt's technique **Level: Easy**

What are the dimensional parameters that control the stablity of floating bodies using Nusselt's technique assuming the body are with uniform density.

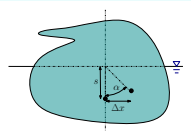


Fig. 12.3 – Floating body showing alpha and other dimensions.

Solution

The governing equation for the stability of floating body is

$$\theta = \alpha \tag{12.13}$$

where θ is the arbitrary turning angle and α is the angle resulting from change of the centroid of submerge volume due to the change in θ. The relationship between the various geometrical parameters is determined connection according to Eq. (12.13). That relationship requires look at the component of triangle at Fig. 12.3. The base of the triangle is determined by

$$\Delta x = x_n - x_0 = \frac{V}{V_0}\left(x_a - x_r\right) \tag{12..l}$$

End of Ex. 12.5

and x_a and x_r is related to ratio

$$x_a = \frac{\int x\, dV}{\int dV} = \frac{\int x(x\tan\theta)d(x)\, dx}{V} = \frac{\tan\theta \int x^2 \overbrace{d(x)\, dx}^{dA}}{V} \tag{12..m}$$

or using the definition of moment of inertia Eq. (12..m) can be transferred into

$$x_a = \frac{\tan\theta\, I_{xx}}{V} \tag{12..n}$$

After the opposite side calculation, adjacent side ($\boldsymbol{G\mathcal{B}}$ center of ($\boldsymbol{\mathcal{B}}$) buoyancy ($\boldsymbol{G}$) center of gravity) thus ($\boldsymbol{G}$-$\boldsymbol{\mathcal{B}}$). The weight can be estimated as $m = \rho_\ell\, A\boldsymbol{\mathcal{B}} = \rho_s\, A\boldsymbol{G}$. When A is typical cross section, thus

$$\boldsymbol{G\mathcal{B}} = \boldsymbol{G}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{12..o}$$

Combining equations (12..n) and (12..o) results in

$$\alpha = \frac{\dfrac{\tan\theta\, I_{xx}}{V}}{\boldsymbol{G}\left(1 - \frac{\rho_s}{\rho_\ell}\right)} \tag{12..p}$$

Observation of Eq. (12..p) so dimensional group $\frac{\rho_s}{\rho_\ell}$, and $\frac{I_{xx}}{\boldsymbol{G}V}$ While the analysis was clumsy and rough it provides dimensionless parameters while Buckingham's method fails dramatically.

12.3 Summary

The two dimensional analysis methods or approaches were presented in this chapter. Buckingham's π technique is a quick "fix approach" which allow rough estimates and relationship between model and prototype. Nusselt's approach provides an heavy duty approach to examine what dimensionless parameters effect the problem. It can be shown that these two techniques in some situations provide almost similar solution. In other cases, these technique proves different and even conflicting results. The dimensional analysis technique provides a way to simplify models (solving the governing equation by experimental means) and to predict effecting parameters.

13
Experiments for Stability

13.1 Introduction

This chapter intended for students who taking a stability class or fluid mechanics beside professionals in marine industry. This chapter is intended to be supplemental to the academic material but in other situations it could be a stand alone material. While stability has mainly theoretical subjects. Yet, there are elements that should provide the student with a hands–on experience. For this reason, several experiments are designed to be performed by the instructor or by the students to provide the "missing" intuition. The floating body stability is not intuitive as the stability of placing body on solid surface and requires demonstration for various aspects. Even professionals could benefits from these experiments due to the fact that many of these aspects are totally innovative.

The first experiment deals with the stability diagram of floating bodies. Every geometry of floating body has its own stability diagram and diagram demonstrates three zones: stable, unstable, and neutral situations. The second experiment deals again with stability diagram but for different geometries. The third experiment the measurements of $\boldsymbol{GM}$ of the cylinder are carried out. The natural frequency of the floating body is measured for constant cross. The change of the rotation location is estimated. The principle axes experiment in which general shape of the "preferred" axes of rotation is examined.

The every experiment will be written as a separated document (that is, start from a new page) and appear in this collection.

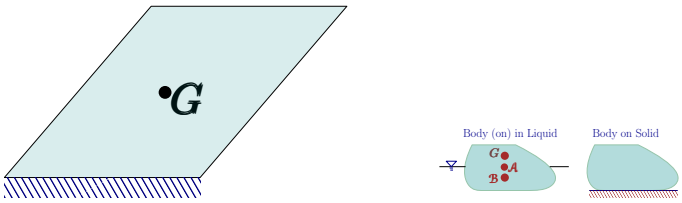
13.2 Stability Points of Various Bodies

13.2.1 Concept

Every body has a stability points (angles or positions) either on solid surface or in liquid (while floating) for which the body is stable. The number of these points and their angle depends on the geometry of the body. The intention of this experiment is to build intuition in regards to the stability points and demonstrate the unpredictability of these angles. This discussion is for the uniform density bodies.

13.2.2 Description

The existences of stability points is a fundamental concept in the stability analysis which could be carried out by high school students , university students and by professionals. The experiment is designed to demonstrate the existence of clear points (angles) in which the body is stable around. At stability points, the body is stable while at other angles the body is unstable. These points are different from the angles when the body is at rest on solid surface. The neutral stability is also demonstrated in this experiment.



(a) G over the base: stable (b) G out of the base: unstable

Fig. 13.1 – Body on solid surface stable and unstable.

These bodies shown in the Fig. 13.1 have different stability points on liquid. The body depicted in Fig. 13.1a has four point on solid and floating on liquid. While the body depicted in Fig. 13.1a has two points but on liquid these point are at different angles.

13.2.3 Apparatus

The experiment is based on observation and not measurement and is relatively simple to carry out. Yet, there are several issues to be pointed out, placing body on solid surface compared to placing the body in liquid, estimating the number stable points (again should be compared to solid), comparison change of one parameter of the geometry, and change of density ratio. A container (preferred transparent) with an ability to contain enough liquid in which the floating bodies can to be examined. Several bodies: extruded squire, extruded circular, extruded rectangular (with different ratios). It also should include a body similar to the one depicted Fig. 13.1b and one similar to one depicted in Fig. 13.1a. The same bodies should be made out two different materials (light and heaver like a styrofoam, and wood).

13.2.4 Procedure

State what are your predicting angles prior the experiments and tabulate the prediction in the following tables.

Table 13.1 – Angle of Stable points Styrofoam

	Solid		Liquid	
Shape	**Prediction**	**Actual**	Prediction	Actual
square				
rec ratio 1:1.1				
rec ratio 1:1.2				
rec ratio 1:1.3				
rec ratio 1:1.4				
rec ratio 1:1.5				
rec ratio 1:1.6				
rec ratio 1:2				
cylinder				
ball				

Table 13.2 – Angle of Stable points Oak

	Solid		Liquid	
Shape	**Prediction**	**Actual**	Prediction	Actual
square				
rec ratio 1:1.1				
rec ratio 1:1.2				
rec ratio 1:1.3				
rec ratio 1:1.4				

Continued on next page

Table 13.2 – Angle of stable point Oak (continue)

Shape	Prediction	Actual	Prediction	Actual
rec ratio 1:1.5				
rec ratio 1:1.6				
rec ratio 1:2				
cylinder				
ball				

Place the bodies on solid surface and observed the results. Place the cylinder in different angle and observed if the cylinder change or stay in the placed angle. Repeat the same for the ball.

13.2.5 Data Analysis

Calculate the density of the oak and styrofoam. Using the information build a stability diagram for the rectangular shape.

13.2.6 Additional Questions

1. Is possible to build the same diagram for triangle?
2. Is possible to build this diagram for other shape? what are the requirements for this diagram on the geometry?

13.3 Cylinder Stability Diagram

13.3.1 Concept

This laboratory experiment is an exercise in stability. It is designed to demonstrate the stability of floating bodies and to familiarize the student with the concepts of buoyancy, neutral and stable and unstable situations, and stability dome. Every floating body has a stability diagram which is depicting stability for certain angle. The vertical position (angle) is often the most alluring for many applications. For example, rectangle in a vertical position is useful for marine applications.

13.3.2 Description

The stability of the floating body for specific angle has what known as the stability dome. The floating body buoyancy centroid, $\mathfrak{B}$, depends on the body shape and on its orientation.

A floating body has several stable positions (angles). The vertical position mimic to some extent the shape of ship. A position can be stable under a certain density ratio range. The stability dome is introduced via the change of the density ratio. In other words, the experiment is designed to investigate the change density effect on the stability. The change of the density effect is obtained by adding sand (or liquid) to the cylinder. The additional material add weight and hence the effective density c hanges. The schematic of the experiment is shown diagram Fig. 13.2.

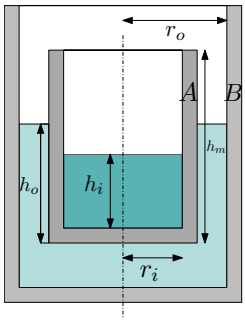


Fig. 13.2 – Schematic the stability dome Apparatus.

Note that the gravity centroid changes also during filing process and it is accounted by introducing the effective density ratio. The effective density concept is emulate body with a constant density.

This experimental is verification of the theory presented in this textbook. The dome shown in stability map represents the limits of floating body stability. The effective density of the floating body (ρ_s) is controlled by the amount liquid in the cylinder. A discussion about the effective density is below. Thus, the density ratio can be varied by filling the cylindrical container with liquid or sand.

The core issue of the floating body stability is the locations of gravity centroid and buoyancy centroid. The gravity is fixed in case of uniformed density body. In

this case the gravity depends on the liquid level in the in the inner container (A). The effective density ratio (or the simulated solid body density) is changed by amount liquid poured into cylinder. As opposed to the fix case the buoyancy centroid can be calculated from the middle of the height of the inner cylinder (regardless of the thickness of the container.)

13.3.3 Apparatus

There are three components of the experimental set-up shown in Fig. 13.2:

1. Large vertically-standing cylinder containing liquid (fresh water) denoted in figure as B
2. Small cylinder with a detachable cap at one end denoted in figure as A
3. weighing scale

Insert the inner container to the large container empty. Fill the inner container with small incremental amount of liquid (see Fig. 13.2). Measure the height of the filling the liquid and records the mount. Check if the inner container is stable.

13.4 The procedure

1. Measure the weight the inner container and indicate it on the report
2. Fill the exterior container with liquid (fresh water or oil, etc)
3. Inserted the inner container into the larger container
4. Measure the thickness inner container wall
5. Measure and record the height h_o. This height is the initial height, h_{o0}.
6. Check if the cylinder is stable and record this information
7. Add small amount of liquid into the inner container.
8. Messure h_i and h_o
9. Go item 6 if the inner cylinder is not full
10. If the inner cylinder is full, it is the end of the process.

13.4.1 Data Analysis

The data analysis is made from two sections: the initial and the process parts. The initial part deals with calibrations.

Initial

1. Calculate the density of the liquid using the weight
2. Use the value of h_{o0} to calculate and verify the weight of the empty inner cylinder, w_{i0}.

$$w_{10} = h_2 \, \pi \, {r_1}^2 \, \rho_\ell \tag{13.1}$$

3. Calculate the initial gravity location of the empty inner container, (assuming thin walls)

$$\boldsymbol{G}_0 = \frac{{h_m}^2 \, \pi \, r_o \, t}{\pi \, {r_o}^2 \, t + 2 \, h_m \, \pi \, r_o \, t} = \frac{{h_m}^2}{r_o + 2 \, h_m} \tag{13.2}$$

4. Calculate the buoyancy centroid using $\boldsymbol{\mathcal{B}}_0 = \frac{h_o}{2}$

Process

Repeat the following steps for each additional liquid amount:

5. Calculate the buoyancy centroid as $\boldsymbol{\mathcal{B}} = h_o/2$
6. Calculate the effective gravity centroid by

$$\boldsymbol{G}_{eff} = \frac{w_{i0} \, \boldsymbol{G}_0 + 0.5 \, {h_i}^2 \, \pi \, {r_i}^2 \, g \, \rho_\ell}{w_{i0} + h_i \, \pi \, {r_i}^2 \, g \, \rho_\ell} \tag{13.3}$$

7. Compare $\boldsymbol{G}_{eff}$ and h_B. If $\boldsymbol{G}_{eff} > \boldsymbol{\mathcal{B}}$ the experiment is not valid (it is not addressed the case of $\boldsymbol{G}$ over/above $\boldsymbol{\mathcal{B}}$) for uniform density. This data point should be ignored. Continue the next data point.
8. The density ratio can can be calculated by

$$\rho_s = \frac{2 \, h_G}{h_o} \, \rho_\ell \tag{13.4}$$

9. Draw stability dome for the inner container.

Table 13.3 – Table for the calculating various value

index	h_o	h_i	$\boldsymbol{\mathcal{B}}$	$\boldsymbol{G}_{eff}$	ρ_s
1					

Continued on next page

Table 13.3 – Table for the calculating various values (continue)

$index$	h_0	h_i	$\mathcal{B}$	G_{eff}	ρ_s
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

13.4.2 Additional Questions

Questions to the students:

1. Under what conditions the fact that the liquid (or other moving surfaces) is not relevant to the stability analysis?
2. Show that Eq. (13.3) the density of the liquid is irrelevant. Simplify the equation.
3. Explain how Eq. (13.3) was constructed.
4. What is the physical limit of this experiment. Why the experiment is limited?

13.5 $\boldsymbol{GM}$ experiments

13.5.1 Concept

Simply stating, a body is stable if a small rotation is applied and body return to its original state. In the old method is done by measuring the $\boldsymbol{GM}$ as a marker of the stability. The body is stable when $\boldsymbol{GM}$ is positive. Note the $\boldsymbol{GM}$ is monotonic but not a linear function, that is, $\boldsymbol{GM}$=2[m] is more than 1[m] but not double in the "stability."

13.5.2 Description

In this experiment, the stability marker, $\boldsymbol{GM}$, is obtained for specific geometry. In doing so the concepts of stability, and metacenter are introduced. This concept was discussed in chapter 9 and here it is verified. Due to fact this material could be used as a stand alone this description is added.

The buoyancy centroid depends on submerged geometry. Assuming quasi steady state, that is the body in equilibrium with the liquid in the z direction and, forces act on the body gravity and buoyancy are equal. However, the only deviation from the quasi steady state is in the roll (change in angle of the body in the yz plane as defined in chapter 2).

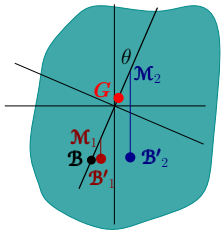


Fig. 13.3 – Genral change of $\mathcal{B}$ with two possibilities.

Buoyancy depends on the body shape and orientation (angle). When the body is tilted in a small amount the buoyancy centroid moves as well. Thus, the buoyancy centroid movement is such that $\mathcal{B}$ moves to two possible zones with a border (the border is not shown explicitly) as shown in Fig. 13.3. The right side is stable (blue color) and the unstable (brown color). The border is made by a circular body which is neutral stability (under the $\boldsymbol{G}$ point). The point of intersection of the lines of vertical line from the new buoyancy centroid with the previous vertical line that goes through the $\mathcal{B}$ is referred as the metacenter ($\boldsymbol{M}$) and the distance between the center of gravity ($\boldsymbol{G}$) and $\boldsymbol{M}$, is referred as the metacentric height ($\boldsymbol{GM}$, see Fig. 13.3).

The expression for the metacentric height $\boldsymbol{GM}$ is

$$GM = \frac{I_{xx}}{V_0} - \boldsymbol{G\mathcal{B}} \tag{13.5}$$

where I_{xx} is the floating body moment of inertia at the liquid line, $\boldsymbol{G\mathcal{B}}$ is the distance between the gravity centroid and the old buoyancy centroid and V_0 is volume of the displaced liquid. For stability the metacentric height $\boldsymbol{GM}$ must be positive. Stability (restoring force) increases with increasing $\boldsymbol{GM}$.

In this experiment, the mission to estimate the metacentric height for several floating bodies.

13.5.3 Rig

There are three components of the experimental set-up:

1. A large pot (preferred in a cylinder shape) containing liquid (the liquid can be salt water, fresh water, oil, etc.),
2. scale,
3. a additional sand or liquid,
4. relatively smaller pot to be fitted into the first pot (thus it should be made from cylindrical shape or, at least, both can be any shape that fitted into each other proffered to be in a form that easy to calculate (or pre–calibrated) the volume), and,
5. and a metric balance to weigh the cylinders.

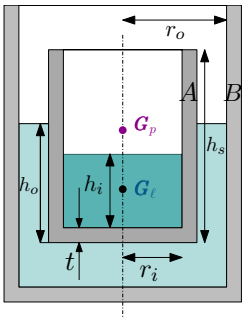


Fig. 13.4 – Apparatus for $\boldsymbol{GM}$.

13.5.4 Procedure

1. Measure the weigh of the small pot make it w_s
2. Place the small empty cylinder into the large vertical cylinder filled with liquid and observe stability (whether or not the cylinder is stable).
3. Add a small amount of sand/liquid into the small cylinder and describe if the small cylinder is unstable. Measure the distance of the small cylinder is submerged, h_o, and then measure the height of liquid/sand, h_i.
4. Continue adding sand until the small cylinder remains vertical, i.e., stable. Measure the submerged amount of cylinder. (This is the stability boundary)
5. Continue the process until the cylinder become stable again. Mark that point as the second stability boundary on the $\boldsymbol{GM}$ diagram or the stability diagram.
6. Remove the small cylinder and measure its weight. Also measure the height of sand and record your results on the data sheet.

13.5.5 Measurements

According to procedures described above, measure the following quantities:

Table 13.4 – Measurement table

Measurement Table					
Case	**Mass** (gm)	$h[cm]$	$h_i[cm]$		
Unstable					
Stable					

13.5.6 Data Analysis

Fig. 13.4 shows apparatus that used for this experiment which a cylinder (pot) inside another cylinder (pot). The inside cylinder is filled by material. The cylinder floats in a liquid with density of ρ_ℓ, as it marked in schematic. The buoyancy centroid is at $\mathcal{B} = h_o/2$. The center of the actual gravity centroid, $\boldsymbol{G}$, is given by

$$\boldsymbol{G} = \frac{w_i\,(h_i/2 + t) + w_A\,h_s}{w_i + w_A} \tag{13.6}$$

The metacenter is given (assuming that it is cylinder) by

$$\boldsymbol{GM} = \frac{\pi\,{r_i}^4}{\pi\,{r_i}^2} - (\boldsymbol{G} - \mathcal{B}) \tag{13.7}$$

Table 13.5 – Trapezoid properties and equations

The calcualted values							
h_o		h_i		$\mathcal{B}$		$\boldsymbol{G}$		$\boldsymbol{GM}$

Continued on next page

Table 13.5 – Basic Cone properties (continue)

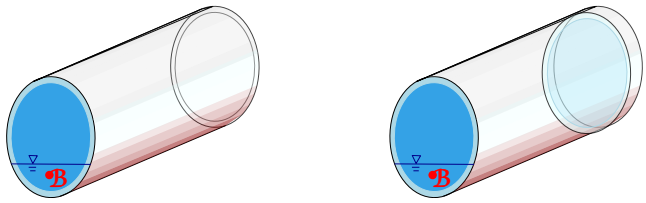
The calcualted values							
h_o		h_i		$\mathcal{B}$		G		$G\mathcal{M}$

13.5.7 Additional Questions

1. In the vertical position, how many stable positions are there?

2. Is there any other layout or angles of the cylinder beside the vertical position that body is in stable positions? (hint, check the photos in this book) When the cylinder is layout in the liquid as shown in Fig. 13.5 are considered as neutral stable (that is, consider the symmetrical shape and its orientation in x coordinate as defined for ships.). What are the conditions that these cylinders for this to be true (hint, see the discussion about it in book). Is the cap at the cylinder end affects this neutral symmetry.

3. Consider a hollow circular cylinder of homogeneous material of specific gravity, $s = 0.5$, length, L, and diameter, D, stable at the position shown in Fig. 3. Find the formulas for the immersion of the cylinder h at this position.



(a) hollow cylinder without cap (b) hollow cylinder with cap

Fig. 13.5 – cylinder laying on liquid with and without cap.

4. Consider a hollow cylinder made with two plates of thickness $t = \xi r$, and the density of the solid is $a\,\rho_\ell$. Assume the cylinder is filled with air which mass can be ignored. what the range of ξ that this cylinder will float?

5. A hollow cylinder is with wall of uniform thickness, t is floating vertically in liquid. The ratio of the height and ratios of cylinder are L and r respectively. Determine the ranges of r/L for which the positions of the vertical cylinder is stable. The density of the liquid is ρ_ℓ.

13.5.8 References

Meta

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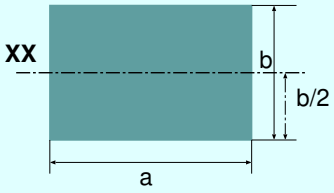
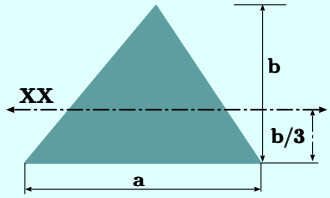
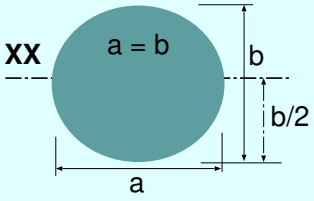
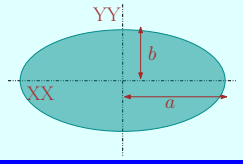
Meta End

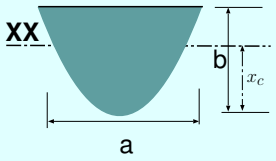
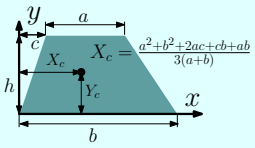
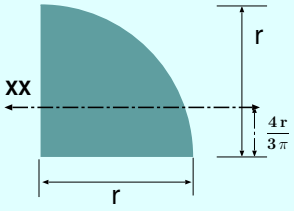
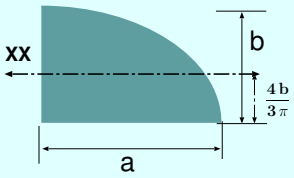
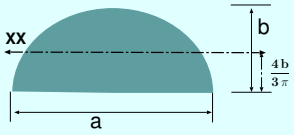
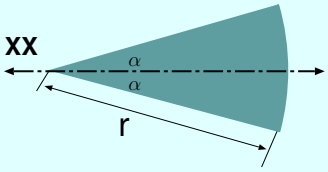
Data

Table 6 – Moment of Ineria and Other Data

Shape Name	Geometry	Centroid	Area	*I*
Rectangle		$\frac{b}{2}\,;\frac{a}{2}$	$a\,b$	$\frac{ab^3}{12}$
Triangle		$\frac{a}{3}$	$\frac{a\,b}{3}$	$\frac{ab^3}{36}$
Circle		$\frac{b}{2}$	$\frac{\pi\,b^2}{4}$	$\frac{\pi b^4}{64}$
Ellipse		$\frac{a}{2}\;\frac{b}{2}$	$\frac{\pi\,ab}{4}$	$\frac{ab^3}{64}$

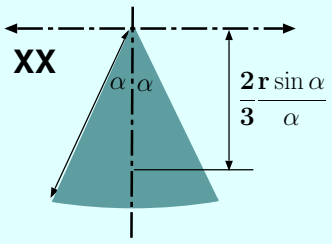
Continued on next page

Table 6 – Moment of inertia (continue)

Name	Geometry	Centroid	Area	I
$y = \alpha x^2$ **Parabola**		$\frac{3\,a\,b}{15\,\alpha - 5}$	$\frac{6\alpha-2}{3} \times \left(\frac{b}{\alpha}\right)^{\frac{3}{2}}$	$\frac{\sqrt{b}\left(20\,b^3 - 14\,b^2\right)}{35\,\sqrt{\alpha}}$
Trapezoid		$Y_c = \frac{h(2a+b)}{3(a+b)}$	$\frac{h(a+b)}{2}$	$\frac{h^3(3a+b)}{12}$
Quadrant of Circle		$\frac{4\,r}{3\,\pi}$	$\frac{\pi\,r^2}{4}$	$r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$
Ellipsoidal Quadrant		$\frac{4\,b}{3\,\pi}$	$\frac{\pi\,a\,b}{4}$	$a\,b^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$
Half of Elliptic		$\frac{4\,b}{3\,\pi}$	$\frac{\pi\,a\,b}{4}$	$a\,b^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$
Circular Sector		0	$2\alpha\,r^2$	$\frac{r^4}{4}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)$

Continued on next page

Table 6 – Moment of inertia (continue)

Name	Geometry	Centroid	Area	I
Circular Sector		$\frac{2}{3}\frac{r\,\sin\alpha}{\alpha}$	$2\alpha\, r^2$	$I_{x'x'} =$ $\frac{r^4}{4}\left(\alpha+\frac{1}{2}\sin 2\alpha\right)$

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Index of Subjects

π–theory, 156

Added mass, 125
Added moment of inertia, 8
Added moment of inertia of the ship, 139
Approximation of centroid, 29
Archimedes principle, 71
Assumption of Constant Mass, 144

Bilge, 13, 16
Block coefficient, 12
Boiling water, 136
Buckingham's theorem, 164
Buoyancy, 72
buoyancy, 71
Buoyancy centroid, 90
Buoyancy explanation, 74

Camber, 16
Centerid
    calcualtions
        circular, 109
Centroid
    Circular, 109
Centroid in 3D, 37
Change in x-direction, 40
Change in y-direction, 40
Change of rotation hinge, 144
Change of Stability with density, 2

Dampingless equation, 139
Deadweight, 12
Definition of Ship Body, 9
Deviation factors, 15
Deviation offset, 14
Dimensional analysis, 155
    Basic units, 157
    Parameters, 161
Dimensionally compartmental, 5
Direct calclation of segment Centroid, 109
Direct Examination, 6, 113
Direct examination method, 5
Displaced liquid, 90
Double Pendulum, 150
Double pendulum, vi
    circular mass distribution, 150
    circular single mass, 150
    linear mass mass distribution, 150
    linear mass single mass, 150
Drowning of Vesa, 3
Dynamics similarity, 168

Even keel, 13

Free falling, 78
freeboard, 10
Froude Displacement Coefficient, 15

Geometric similarity, 167

Heave, 136
Heaving, 16
Heel, 13
hinge line, 144

Indexical form, 163

Kinematic similarity, 167
King Gustavus, Adolphus, 3

Lagrangian function, 151
Lautrup's diagram, 5
Light Displacement, 11
Line density, 28
List, 136
Load Displacement, 11
Load limitation, 1
Load lines, 10
Lost containers, 6

Mass
    area centroid, 27
Mathieu's equation, 142
Maximum section Area Coefficient, 12
Metacenter, 72
    point, 113
Metacenter method, 113
Metacentric Height, 126
metacentric height, 179
Moment of Inertia, 49
    2D,3D, 54
    Area, 49, 50
    Definition, 49
    Mass, 49
    Partial, 52
    Transformation, 51
    Zero, 143
Moment of inertia
    cuboid, 140
Money and buoyancy, 71

Natural frequency, 151
Neutral moment
    Zero moment, 89
Neutral stable, 89

Parallel axis theorem, 51
Pendulum action, 125
Pendulum problem, 162
Pitch, 13
Point force, 27
Port side, 10
positive trim, 14
Potential energy examination, 72
Potential Energy Stability, 4
Potential stability method, 4
Pressure center, 69, 70
Pressure line, 70
Prismatic coefficient, 12, 15
Product of Inertia, 57
    Area, 49

Quasi–equilibrium, 136

Radius of gyration, 49
Relative change in center of mass, 27
Roll, 13
Rolling, 135
Rotation hinge
    Pendulum, 136
Runge Kutta method, 151

Salinity effect, 10
Shifting Moment of Inertia, 96
Ship bean, 10
Ship Length, 14
Ship's list, 14
Ship's zigzag movement, 14
Similitude, 157, 167
Specific density/gravity, 3
Stability
    Prerequisite Condition, 92
stability analysis, 71
    cubic, 93
Stability Diagram, 2
Stability diagram, 5
Stability dome, 5
Stability on solids, 2
Stability point, 1
Stability points/angles, 2
Stability usage for centroid, 108
Stability zones, 139
Starboard, 10
Surge, 16
surge, 136
Sway, 16, 136

Temperature effect, 10
Terminology, 9
Tonne Per Centimeter, 11

Total moment, 64
Transfer mechanism, 148
    Linear to Linear, 149
    Linear to Rotation, 149
    Rotation to Rotation, 148
Transfer of axis theorem, 57
Trim, 13, 136

Vectors, 19
Vectors Algebra, 20
Vertical action line, 5
Vertical Stability, 1

What is stability issue?, 1

Index of Authors

Abolhassani, Mohammad, 4
Acco Nautical College, 71
Archimedes, 3, 71

Bar-Meir, Genick, 6
Berrett, xlvi
Biran, xlvi
Blanke, Mogens, 7
Bouguer, Pierre, 4
Buckingham, 156, 157
Bulian, Gabriele, 7, 139

Erdös, Paul, 4
Euler, Leonhard, 4

Fourier Jean B. J., 157
France, William N, 138
Francescutto, Alberto, 7
Froude, William, 157
Fucile, F, 7

Galeazzi, Roberto, 7
Gualeni, Paola, 7

Hamed, YS, 6
Herndon, Roy C, 4
Hoste, Paul, 3
Huygens, Christiaan, 3, 4

Jovanoski, Zlatko, 139

La Croix, 4
Landweber, L, 143
Lautrup, Benny, 5

Martinelli, 156
Maxwell, 157
Miloh Touvia, 143

Newman, John Nicholas, 7
Newton, 157
Nusselt, Ernst Kraft Wilhelm, 156, 157

Petacco, Nicola, 7
Poulsen, Niels K, 7

Rawson, Kenneth John, xlv
Rayleigh, 157
Remola, Adriana Oliva, 143
Reynolds, Osborne, 157
Riabouchunsky, 157

Sayed, M, 6
Scarpa, S, 7
Schibler, Gérard, 4
Schmidt, 156
Serra, A, 7
Shin, Young S, 138
Smeaton, John, 157
Spyrou, KJ, 139
Stevin, Simon, 3

Taylan, Metin, 137
Thompson, JMT, 139
Touvia Miloh, 153
Tupper, Eric Charles, xlv

Van Laarhoven, BJH, 138
Vaschy, Aiméem, 157